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Statistical Analysis of Electrostatic Spark Ignition of Lean $H_2/O_2/Ar$ Mixtures☆

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Abstract

The concept of minimum ignition energy (MIE) has traditionally formed the basis for studying ignition hazards of fuels. However, the viewpoint of ignition as a statistical phenomenon appears to be more consistent with the inherent variability in engineering test data. We have developed a very low-energy capacitive spark ignition system to produce short sparks with fixed lengths of 1 to 2 mm, and the ignition system is used to perform spark ignition tests using a range of spark energies in lean hydrogen-oxygen-argon test mixtures used in aviation safety testing. The test results are analyzed using statistical tools to obtain probability distributions for ignition versus spark energy. A second low-energy spark ignition system was also developed to generate longer sparks with varying lengths up to 10 mm. A second set of ignition tests is performed in one of the test mixtures using a range of spark energies and spark lengths. The results are analyzed to obtain a probability distribution for ignition versus the spark energy per unit spark length. Preliminary results show that a single threshold MIE value does not exist, but rather that ignition is statistical in nature and highly dependent on mixture composition and spark length.

Keywords: combustion, spark ignition, statistical analysis

1. Introduction

Determining the risk of accidental ignition of flammable mixtures is a topic of tremendous importance in industry and in aviation safety. As progress is made world-wide in using hydrogen as an energy source, it is particularly important to address the safety hazards posed by ignition of hydrogen from leaks in storage vessels or in duct outlets [1, 2]. Extensive work has been done [3–5] to determine the flamma-

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bility limits of various fuels in terms of mixture composition. These studies were all performed using a very high-energy ignition source that is assumed strong enough to ignite any mixture with composition within the flammability limits. When examining the amount of energy required to ignite a mixture within the flammability limits, the concept of a threshold minimum ignition energy (MIE) value has traditionally formed the basis in combustion science for studying ignition hazards of fuels. If an ignition source is not strong enough, or is below the minimum ignition energy (MIE) of the particular mixture, the mixture will not ignite. Standard test methods for determining the MIE have been developed [5, 6] which use a capacitive spark discharge for the ignition source. The MIE is determined from the energy stored in a capacitor at a known voltage that is then discharged through a specified gap. The pioneering work using this ignition method to determine MIE values was done at the Bureau of Mines in the 1940s by Guest, Blanc, Lewis, and von Elbe [7]. They obtained MIE data for many different fuels and mixture compositions, and this data is still extensively cited in the literature and ignition handbooks. This technique is also used to study ignition hazards in the aviation industry and standardized testing is outlined to determine the MIE of aviation test fuels [8, 9]. Since the work at the Bureau of Mines, many authors have proposed improvements on the technique for determining MIE using capacitive spark discharge, most recently Ono et al. [10] and Randeberg et al. [11]. While several authors have worked on developing numerical models of spark ignition, e.g. [12–14], predicting ignition remains primarily an experimental issue.

The view of ignition where the MIE is considered to be a single threshold value is the traditional viewpoint in combustion science [7] and extensive tabulations of this kind of MIE data are available [5, 6]. In recent years however, particularly in the aviation safety industry, the viewpoint has shifted to ignition being a statistical phenomenon, an approach that is more consistent with the inherent variability in engineering test data. Simple statistical methods have been applied to Jet A spark ignition tests performed by Lee and Shepherd at the California Institute of Technology [15], and Colwell and Reza have applied statistical analysis to hot surface ignition data for liquid fuels [16]. However, there has been very little work done [9] on the statistical nature of spark ignition of hydrogen-based mixtures, which are often used as test mixtures in aviation safety certification. To determine the statistical nature of the data, the experimental variability must be minimized and quantified, and an ignition source is required that is well-controlled and fully characterized.

In ignition testing there are many uncontrolled sources of variability in the experiment itself separate from the ignition energy. These uncertainties can lead to inaccurate test results and the appearance of variability in the results that has no correlation with the ignition energy. One major cause of variability in the test results is uncertainty in the mixture composition. Changes in composition lead to changes in combustion characteristics and MIE values. Therefore it is extremely important to precisely control and accurately measure composition during ignition experiments. Another cause of variability is the degree of turbulence near the ignition source, as the process of flame initiation and propagation can be affected by pre-existing turbulence. Finally, a third important source of variability in the test data is the method used to detect ignition. If the detection method is unreliable or unsuitable for the combustion characteristics of the mixture being tested, a given ignition energy may be perceived as not igniting a mixture when in fact combustion did occur.

Previous work has been done to assess these three sources of variability in tests involving lean hydrogen-oxygen-argon aviation test mixtures and to propose test methods to minimize these uncertainties [17]. Another source of variability is assuming the spark energy is equal to the stored energy in the capacitor ($CV^2/2$) when in fact it is only a fraction of the stored energy. To address this issue, we take spark current measurements and estimate the energy for each spark. The sources of experimental uncertainty are not limited to the four described here, but these are major contributors to variability in the data that is unrelated to the ignition source. It is therefore necessary to quantify and minimize the uncertainties from these four contributing sources before the variability of ignition with respect to ignition source energy can be examined.

In the present work, statistical tools were adapted to use with ignition tests to describe the test results in terms of probability distributions and confidence intervals. Capacitive discharge systems were developed to produce very low-energy (50 μJ to 1 mJ) sparks for both fixed spark lengths and variable spark lengths. Ignition tests were performed in lean hydrogen-based test mixtures used in aviation safety certification, and the results were analyzed using the statistical tools to investigate the variability of ignition versus spark energy and spark energy density.

2. Statistical Analysis of Ignition Test Data

Given a set of ignition test data points, the goal of the statistical analysis is to derive a probability distribution for the probability of a "go" result (ignition) versus stimulus level (i.e., spark energy). In this work, the logistic regression method [18, 19] is used to calculate a cumulative probability distribution for the ignition data. The logistic distribution was also used by Colwell and Reza to analyze hot surface ignition [16], and the logistic regression method is computationally simple because likelihood-based inference methods can be used to make assumptions about the parameters.

A binary outcome model is used for spark ignition tests with a binary result y, where $y = 1$ for a "go" (ignition) and $y = 0$ for a "no go" (no ignition) for a given stimulus level x (spark energy or energy density). The cumulative probability distribution for a "go" at stimulus level x can be defined as

$$P(x) = \text{Probability}\,(y = 1; x)\ . \tag{1}$$

All the stimulus levels and the binary results for the n tests are represented collectively using the likelihood function

$$L = \prod_{i=1}^{n} P(x_i)^{y_i}\,(1 - P(x_i))^{1-y_i}\ . \tag{2}$$

The probability $P(x)$ is assumed to be represented with the parametric logistic distribution function

$$P(x) = \frac{1}{1 + \exp(-\beta_0 - \beta_1 x)} \tag{3}$$

where β_0 and β_1 are parameters that are estimated by maximizing the likelihood function (Equation 2). The upper confidence limit (UCL) and lower confidence limit (LCL) for the $100\,(1-\alpha/2)\%$ confidence interval for the percentile x_q can be calculated using the large sample approach for a two-sided interval [19]. The result of this analysis is a cumulative probability distribution for the n spark ignition tests and a confidence envelope on the probability of ignition versus spark energy or energy density. Further details on the statistical methods are presented in [20].

3. Short, Fixed Spark Ignition Testing

3.1. Short Spark Ignition System

To perform ignition testing near the MIE with the ultimate goal of performing statistical analysis on the resulting data, a well-characterized and repeatable low-energy ignition system is required. Therefore, in this work a low-energy capacitive spark ignition system has been designed and constructed to produce short (1 to 2 mm) sparks with energies in the range of 50 μJ to 1 mJ. The discharge circuit is based on the ideas presented by Ono et al. [10]. The basis of the design is a simple capacitive discharge circuit, but many features have been implemented to improve the system performance in terms of reliability, consistency, and repeatability so that the spark energy can be reasonably predicted and measured. A 3-30 pF variable vacuum capacitor is charged by a 0-15 kV high voltage power supply through two 50 GΩ isolation resistors. The voltage is ramped up slowly and when the breakdown voltage of the gap is reached the capacitor discharges across the gap, producing a spark. The breakdown voltage is measured using a high voltage probe and the spark current is measured using a fast current transformer. The circuit is enclosed in an acrylic tube and dry air is circulated through the tube to reduce the humidity. Humidity control is important because moisture can cause the charge to leak from the capacitor. More details on the spark ignition system are presented in [20].

3.2. Estimating Spark Energy

The traditional practice in spark ignition testing is to report the energy stored in the capacitor of the spark circuit rather than the actual energy in the spark that heats the volume of gas to initiate combustion [7]. There are several sources of energy loss, but they are extremely difficult to quantify and depend strongly on the circuit parameters and test methods. In this work we consider only the residual energy in the capacitor after discharge. We approximate the spark energy as the difference between the initial stored energy in the capacitor and the residual energy

$$E_{spark} \approx E_{stored} - E_{residual} \tag{4}$$
$$= \frac{CV_{breakdown}^2}{2} - \frac{Q_{residual}^2}{2C} \,. \tag{5}$$

The total capacitance, C, which includes the contribution of the capacitor and the stray capacitance due to electrical leads and the spark gap, is measured before the test using a precision LCR meter. The voltage on the capacitor at breakdown, $V_{breakdown}$, is measured by the high voltage probe and recorded on the oscilloscope. The residual charge in the capacitor, $Q_{residual}$, is calculated by subtracting the charge delivered in the spark from the original stored charge in the capacitor

$$Q_{residual} = Q_{stored} - Q_{spark} \tag{6}$$
$$= CV_{breakdown} - \int i_{spark}\,(t)\,dt \,. \tag{7}$$

The integral of the spark current is calculated by numerically integrating the waveform from the current transformer.

3.3. Experimental Setup

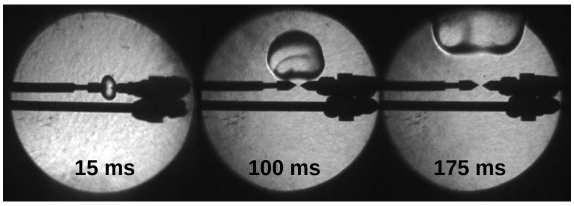
The spark ignition tests are conducted in a closed, rectangular combustion vessel approximately 11.75 liters in volume. The ignition system fixture is mounted in a specialized flange in one of the vessel walls such that the spark gap is near the center of the vessel. Two parallel vessel walls have 1 inch thick glass windows for visualization. A remotely controlled plumbing system is used to evacuate the chamber and accurately fill it with gases using the method of partial pressures. The static pressure is measured by a Heise 901A manometer with a precise digital readout so the gases can be filled to within 0.03% by volume, providing precise control over the mixture composition. To ensure that the mixture is homogeneous, the gases are thoroughly mixed for 3 minutes using a fan mixer inside the vessel. Three methods are used to detect whether or not ignition occurred. One method is measuring the dynamic pressure in the vessel using a piezoresistive pressure transducer. This method provides very sensitive and reliable ignition detection by measuring the transient pressure rise due to combustion, even in cases with modest pressure rise. The second method used to detect ignition is sensing the temperature rise from the combustion with a thermocouple mounted inside the vessel. The third method of ignition detection is visualization of the flame through the vessel windows using a schlieren system and high-speed video camera. While no direct measurement of the degree of turbulence inside the vessel is made, a constant wait time of 3 minutes was observed after turning off the fan mixer to allow any turbulent flucuations to dissipate.

3.4. Results and Discussion

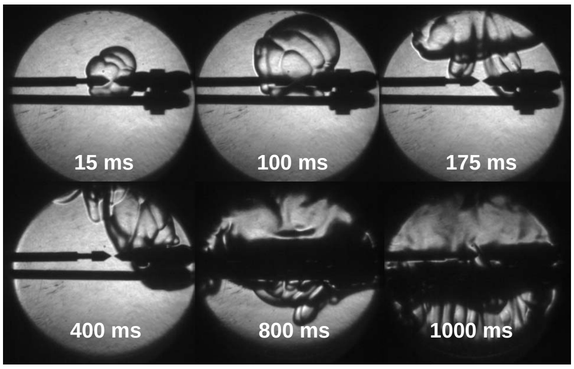
Ignition tests were performed in the aviation test mixture recommended in the ARP testing standards [8], 5% H_2/12% O2/83% Ar, and in two additional mixtures differing only by 1% hydrogen, 6% H_2/12% O2/82% Ar, and 7% H_2/12% O_2/81% Ar. The electrodes used for the short, fixed spark ignition tests were made of tungsten and were conical in shape with a base diameter of 6.35 mm, cone angle of 53°, and a tip radius of 0.8 mm. The spark gap lengths were fixed at 2, 1.5, and 1 mm for the 5%, 6%, and 7% H_2 mixtures, respectively. The characteristics of the flames in the three mixtures were studied using high-speed schlieren visualization, and the ignition test results were analyzed using the statistical tools described in Section 2 to obtain probability distributions for ignition.

3.4.1. Flame Visualization

Images from high-speed schlieren videos of ignition in the three mixtures are shown in Figure 1. Combustion in the 5% H_2 mixture (Figure 1(a)) produces a slow, buoyant flame that propagates upward as well as outward before being extinguished at the top of the vessel. In this case the flame speed is low enough that buoyancy is dominant over the flame front inertia and only a small fraction of the fuel is burned. When the hydrogen concentration is increased by just 1% to 6% H_2 (Figure 1(b)), buoyancy is nearly balanced by the flame inertia. Initially, buoyancy dominates and the flame propagates upwards and the upper flame surface is extinguished at the top of the vessel. However, unlike the 5% H_2 case, the bottom of the flame has enough inertia to continue to propagate downwards, and with assistance from convection induced by the flame, nearly all the fuel is consumed over a long (¿2 s) burn time. When the hydrogen concentration is increased by 1% a second time to 7% H_2 (Figure 1(c)), the flame propagation again changes drastically. In this case, the flame speed is high enough to nearly counteract the buoyancy effects, and the flame propagation is nearly spherical and complete combustion of the fuel is achieved. These results show that the characteristics of the flame propagation for very lean mixtures near the lower flammability limit are extremely sensitive to the mixture composition. This sensitivity to composition and the influence of buoyancy have serious implications for the ARP testing standards. Many tests performed using the ARP standards may not be valid since ignitions near the top of the test vessel may go undetected for very lean mixtures, where flame buoyancy leads to extinction at the top of the vessel.



(a)



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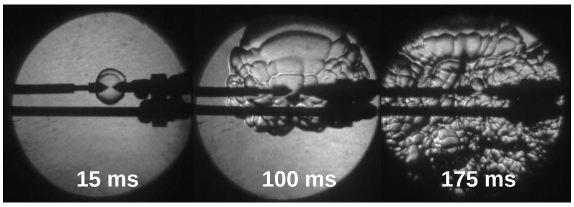

(c)

Figure 1: Images from high-speed schlieren visualization of combustion in (a) the 5% H_2/12% O_2/83% Ar test mixture recommended by the certification standards [8] and mixtures with (b) 6% H_2 and (c) 7% H_2.

3.4.2. Ignition Probability

Ignition tests were performed in each of the three mixtures using a fixed length spark gap (1, 1.5 or 2 mm) and a range of spark energies. The stored energy in the discharge circuit was varied by changing the capacitance and the spark energy was estimated using the method described in Section 3.2. If ignition did occur at a given spark energy, that data point was assigned a result of "1" (a "go"), and if ignition did not occur the result was "0" (a "no go"). These data sets were then evaluated using the statistical tools described in Section 2.0, resulting in probability distributions for ignition versus spark energy for the three test mixtures. As an example, in Figure 2 the ignition test data points are shown with the resulting probability distribution for the 5% H_2 mixture. The "go" and "no go" data points overlap significantly with the highest "no go" occurring for a spark energy of 1022 μJ and the lowest "go" occurring for a spark energy of 790 μJ; this overlap is reflected in the broadness of the probability curve. The probability distributions, 95% confidence intervals, and data overlap region for all three mixtures are shown on the same scale in Figure 3. While the probability distribution for the 5% H_2 mixture is broad, the curves for the 6% and 7% H_2 mixtures are much narrower and closer to representing a threshold MIE value. However, ignition in all three mixtures exhibits considerable statistical variation, suggesting that a statistical approach to analyzing ignition test data is more appropriate than the traditional MIE approach. The statistical analysis also shows significant margin between the median spark energy for ignition because the probability distributions are centered at very different spark energies; the 50% probability of ignition for the 5%, 6%, and 7% H_2 mixtures are 952 μJ, 351 μJ, and 143 μJ, respectively.

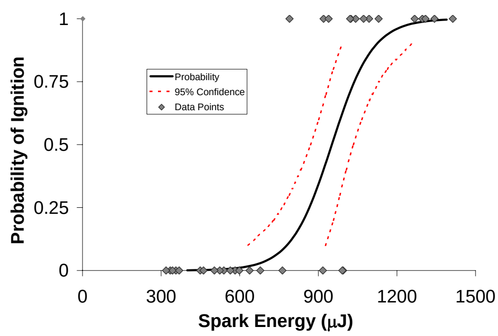

Figure 2: Ignition test data points and resulting probability distribution and 95% confidence intervals for the 5% H_2/12% O_2/83% Ar test mixture.

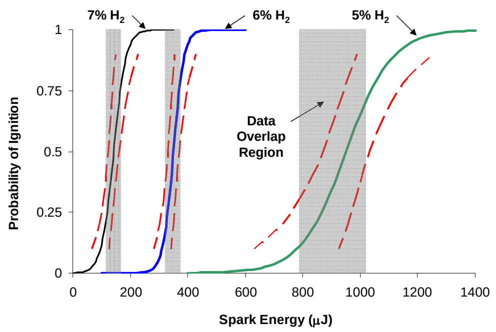

Figure 3: Probability distributions of ignition versus spark energy for the three test mixtures.

The results of the analysis demonstrate clearly that a single threshold MIE value does not exist, but rather that the ignition is statistical in nature and the test results have a significant degree of variability. Furthermore, these results show that for lean hydrogen mixtures near the lower flammability limit, very small changes in the amount of hydrogen not only lead to drastically different flame propagation characteristics but also significant changes in the required ignition energy. Small uncertainties in the test mixture composition or operator adjustments during the test may result in mixtures with ignition thresholds that are substantially different than anticipated, and so extreme caution must be exercised in order to get reliable results with the mixture currently recommended by the ARP (5% H_2). Otherwise, a number of false positives or negatives may occur. Also, the sensitivity of the spark ignition system to humidity indicates that a significant source of variability in the current ARP testing may be due to the lack of humidity control in the commercial test environment.

4. Long Spark Ignition Testing

4.1. Long Spark Ignition System

The ignition tests described in Section 3.4 were performed using fixed sparks with lengths of 1 to 2 mm, but longer sparks are often the possible ignition source of concern in practical applications in aviation and industry. Therefore a second capacitive spark ignition system was developed to generate low-energy sparks with variable lengths on the order of 1 to 10 mm. In this circuit a capacitor is formed by suspending an isolated circular aluminum plate inside the vessel with the vessel itself acting as the ground plate. The capacitance can be varied from approximately 5 to 20 pF by varying the distance between the plate and the vessel wall or by changing the diameter of the plate. The isolated capacitor plate is charged to a high voltage up to 30 kV and a grounded electrode is brought near the electrode on the plate inducing breakdown of the spark gap. The spark length is varied by changing the voltage. By adjusting these variables a range of possible spark energies and spark lengths are possible, from which we can calculate a "spark energy density", or energy per unit spark length. The use of the parameter E/d (energy divided by distance) for characterizing incendivity was proposed by von Pidoll et al. [21]. This ignition system was used to investigate the effect of the spark length on ignition.

4.2. Results and Discussion

A preliminary set of ignition tests in the 6% H_2/12% O_2/82% Ar test mixture were performed using the long spark ignition system over a range of spark energies (100 to 2400 μJ) and spark lengths (1 to 11 mm). The electrodes used in the long spark tests were also made of tungsten with a 6.35 mm base diameter, but the tips are not conical but rather hemispherical with a radius of 3.2 mm so that breakdown at higher voltages can be better controlled. The energy density is obtained by dividing the spark energy by the spark gap length which is measured from schlieren images taken immediately before the gap breakdown. The results were analyzed, using the same statistical tools as employed in the short spark testing, to obtain the probability distribution for ignition versus energy density, shown in Figure 4.

These initial tests demonstrate that the spark energy is not an appropriate quantity for investigating incendivity with sparks of variable lengths. There were many tests with no ignition even though the spark energy was significantly larger than the approximate MIE value (50% probability of ignition) obtained using a fixed spark length. The probability distributions for ignition versus spark energy for the 6% H_2 mixtures using the fixed 1.5 mm sparks and the variabile length sparks are shown in Figure 5. The spark energy with a 50% probability of ignition for the 1.5 mm sparks is 351 μJ, while the 50% probability energy for the variable length (1 to 10 mm) sparks was 745 μJ, nearly twice as large. Therefore, the spark energy cannot be used to compare the fixed length sparks and the variable length sparks. Therefore, the long spark results were analyzed again to obtain a probability distribution for ignition versus spark energy density. The 50th percentile energy density from the 1.5 mm spark tests is 234 μJ/mm (obtained by dividing the 50th percentile energy, 351 μJ, by the spark length of 1.5 mm), while the results of the variable long spark tests give a 50th percentile energy density of 154 μJ/mm, which is much more comparable. The energy density is lower for the long sparks because all the long spark tests where ignition occurred with a spark energy density less than 234 μJ/mm involved spark gaps of 4 mm or longer, so the quenching effect of the electrodes is reduced. Also, in tests with these longer sparks, it can be seen in the schlieren videos that the spark channel is not homogeneous, and that in some cases the ignition kernel forms in only part of the channel where the channel is significantly thicker. In a number of tests, very long sparks with low energy densities still caused ignition due to either a bulging of the spark channel at the cathode where the electrons are bombarding the electrode surface or at some location along the spark channel where the channel is thicker due to an instability in the plasma. It is believed that these bulges in the spark channel have a higher energy density than the rest of the channel, leading to localized ignition kernels.

The ignition test results using sparks of variable length show that, as expected, longer sparks required more energy to ignite the test mixture; even though some sparks had energies far exceeding those of the short, fixed sparks, they failed to cause ignition because the energy density was lower. When the results were analyzed in terms of spark energy density, the results from the tests with the fixed 1.5 mm spark and the results from the variable length spark tests were more comparable. However, the energy density required for ignition with sparks of varying length was lower than in the fixed spark case due to the reduced effect of quenching by the electrodes and inhomogeneity of the spark channel. Therefore, when assessing the risk of ignition, the spark length must be considered in addition to the spark energy. The dependence on the ignition energy threshold on the spark length implies that the entire basis of using MIE values for hazard evaluation is flawed. This consequence is particularly relevant to the problem of evaluating hazards from isolated conductors for which high charging voltages and sparks several millimeters in length occur due to the very low capacitance. Using a fixed ignition threshold energy of 200 μJ, as assumed in the ARP standards, causes significant error in determining the actual hazard.

5. Conclusions

In the current study, ignition tests were performed in lean hydrogen-based mixtures used

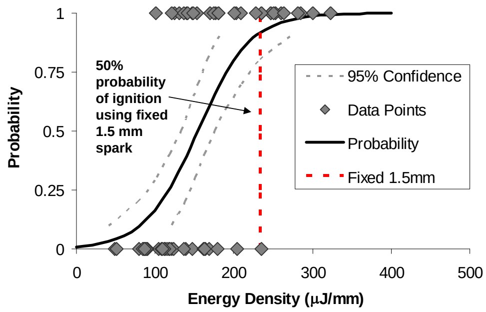

Figure 4: Probability distribution of ignition versus spark energy density (energy per unit spark length) for the 6% H_2 mixture.

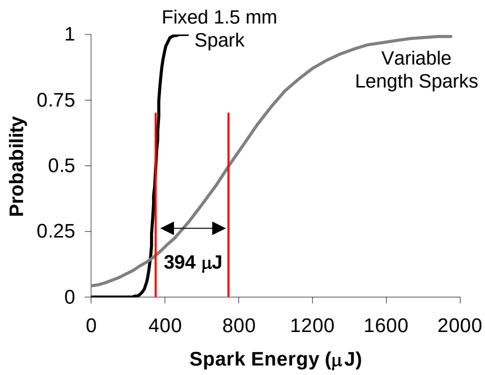

Figure 5: Probability distributions for ignition versus spark energy for the 6% H_2 mixture with the 50th percentiles indicated by the red lines.

in aviation safety certification. Tests were first performed in three mixtures differing by just 1% hydrogen using a fixed length capacitive spark discharge as the ignition source. The results were analyzed using statistical tools and it was demonstrated that a single threshold ignition energy does not exist for a given mixture, but rather that ignition is statistical in nature. It was shown that for very lean mixtures the combustion characteristics and the energy required for ignition are extremely sensitive to the mixture composition, varying drastically between mixtures with 5% H_2 and 7% H_2. Therefore, it is critical to have precise control over the mixture composition and reliable ignition detection when performing tests to determine ignition limits. The experimental method must be designed to minimize experimental variability and the uncertainties must be quantified so that the variability with respect to the ignition process can be isolated.

Further ignition tests were performed in the 6% H_2 mixture using capacitive sparks with lengths varying from 1 mm to 10 mm. The results demonstrated that the energy required for ignition depends strongly on the spark length, and that energy density is a more appropriate quantity than spark energy for quantifying incendivity. However, longer sparks required a lower energy density to cause ignition than shorter sparks, due to the reduced quenching effect and inhomogeneities in the spark channel leading to localized ignition kernels. Therefore, any model of ignition must consider not only the spark energy, but also the spark length, geometry, and electrodynamic effects. The next step in improving aircraft safety certification methods is to define the actual hazard, which is the spark ignition of Jet A (aviation kerosene). Despite the importance of this issue to aviation safety, there are no reliable statistical data available on Jet A spark ignition and so carefully performed spark ignition tests are needed so the hazard posed by Jet A and the hydrogen test mixtures can be compared. The ignition test results and statistical analysis presented in this work represent only the first steps in placing ignition thresholds on a sound statistical basis. Further experimentation with different fuels and a range of electrical discharge parameters is needed.

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[1] G. R. Astbury, S. J. Hawksworth, Spontaneous ignition of hydrogen leaks: A review of postulated mechanisms, Int. J. Hydrog. Energy 32 (2007) 2178–2185.

[2] T. Imamura, T. Mogi, Y. Wada, Control of the ignition possibility of hydrogen by electrostatic discharge at a ventilation duct outlet, Int. J. Hydrog. Energy 34 (2009) 2815–2823.

[3] L. G. Britton, Two hundred years of flammability limits, Process Saf. Prog. 1–11.

[4] H. F. Coward, G. W. Jones, The limits of flammability of gases and vapors, Tech. rep., Bur. Mines, Bull. 503 (1952).

[5] V. Babrauskas, Ignition Handbook: Principles and Applications to Fire Safety Engineering, Fire Investigation, Risk Management and Forensic Science, Fire Science Publishers, Issaquah, WA, 2003.

[6] E. C. Magison, Electrical Equipment in Hazardous Locations, 3rd Edition, Instrument Society of America, 1990.

[7] B. Lewis, G. von Elbe, Combustion, Flames and Explosions of Gases, Academic Press, New York, 1961.

[8] Anonymous, Aerospace recommended practice aircraft lightning test methods, Tech. rep., aerosp. Rep. ARP5416 (2005).

[9] Anonymous, Aircraft fuel system lightning protection design and qualification test procedures development, Tech. rep., U.S. Dept. of Transportation Federal Aviation Administration Report DOT/FAA/CT-94/74 (1994).

[10] R. Ono, N. Masaharu, S. Fujiwara, S. Horiguchi, T. Oda, Gas temperature of capacitive spark discharge in air, J. Appl. Phys. 97 (2005) 123307.

[11] E. Randeberg, W. Olsen, R. K. Eckhoff, A new method for generation of synchronised capacitive sparks of low energy, J. Electrost. 64 (2006) 263–272.

[12] K. Ishii, T. Tsukamoto, Y. Ujiie, M. Kono, Analysis of ignition mechanism of combustible mixtures by composite sparks, Combust. Flame 91 (1992) 153–164.

[13] T. Kravchik, E. Sher, J. B. Heywood, From spark ignition to flame initiation, Combust. Sci. and Technol. 108 (1995) 1–30.

[14] M. Thiele, J. Warnatz, A. Dreizler, S. Lindenmaier, R. Schießl, U. Maas, A. Grant, P. Ewart, Spark ignited hydrogen/air mixtures: Two dimensional detailed modeling and laser based diagnostics, Combust. Flame 128 (2002) 74–87.

[15] J. J. Lee, J. E. Shepherd, Spark energy measurements in jet a part ii, Tech. rep., Grad. Aerosp. Lab., Calif. Inst. Tech., Explos. Dyn. Lab. Rep. FM 99-7 (1999).

[16] J. Colwell, A. Reza, Hot surface ignition of automotive and aviation fluids, Fire Technol. 41 (2005) 105–123.

[17] E. Kwon, S. P. Moffett, J. E. Shepherd, A. C. Day, Combustion characteristics of hydrogen as used in a flammable test mixture, paper PPR-48 (2007).

[18] D. Hosmer, S. Lemeshow, Applied Logistic Regression, John Wiley and Sons, 1989.

[19] W. Taam, Statistical methods in estimating threshold distributions for stress test with single binary outcome, Tech. rep. (2004).

[20] S. P. Moffett, S. G. Bhanderi, J. E. Shepherd, E. Kwon, Investigation of statistical nature of spark ignition, 2007 Fall Meet. West. States Sect. Combust. Inst., Oct. 16-17, Sandia Natl. Lab., Livermore, CA, paper 07F-42 (2007).

[21] U. von Pidoll, E. Brzostek, H.-R. Froechtenigt, Determining the incendivity of electrostatic discharges without explosive gas mixtures, IEE Trans. Ind. Appl. 40 (2004) 1467–1475.