RESPONSE OF PRESSURE TAP

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1 Introduction

In exprimental fluid mechanics one often measures pressure with a sensor that is connected to a point on the surface of a body at which the pressure is to be measured by a thin tube that opens to the surface. It takes time for the volume V of the space that is fed by the tube to fill to the ambient pressure p_0. In gas dynamical applications flows are often started by the passage of a shock wave along the surface. In such situations the initial pressure within the volume V is much smaller than p_0 and the flow through the tube is likely to be initially choked, especially if the tube is short, which is usually the case as the response time must be minimized. This means that we can estimate the initial rate of pressure increase in V.

In the following we first estimate the rate of pressure rise during the choked–flow time. In the phase of the flow in which the pressure ratio is insufficient to choke the flow, we assume that the rate of pressure rise is proportional to the pressure difference in order to obtain an estimate of the pressure as a function of time from the beginning to the steady state.

2 Estimate of initial rate of rise

Assume that the gas in the volume V is spatially uniform at pressure $p(t)$ and density $\rho(t)$. Consider the initial period, during which the flow through the tube of cross–sectional area A is choked, so that the flow velocity through it is the local speed of sound a^*. Let the density at this sonic throat be ρ^*. Then the rate at which mass is added to V is

$$\frac{dm}{dt} = V\frac{d\rho}{dt} = \rho^* a^* A. \tag{1}$$

Now assume that the flow is isentropic, so that

$$\rho^* a^* = \left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(2(\gamma-1))} \rho_0 a_0. \tag{2}$$

Thus,

$$\frac{d\rho}{dt} = \left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(2(\gamma-1))} \rho_0 a_0 \frac{A}{V}. \tag{3}$$

Now it is necessary to make an assumption about the thermodynamic path taken by each element of gas as it proceeds from the throat to the uniform state in V. Though it is certainly not correct, but likely to be approximately true, we make the simplest assumption that the process is isentropic. This means that

$$\frac{dp}{p} = \gamma\frac{d\rho}{\rho}, \tag{4}$$

allowing us to write

$$\rho V \frac{dp}{p} = \gamma \left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(2(\gamma-1))} \rho_0 a_0 A dt. \tag{5}$$

Our assumption about isentropic flow also allows us to write ρ in terms of ρ_0, p_0 and p, giving

$$\frac{dp}{p^{(\gamma-1)/\gamma}} = \gamma \left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(2(\gamma-1))} a_0 p_0^{1/\gamma} \frac{A}{V} dt. \tag{6}$$

Integrating this, we obtain

$$\left(\frac{p}{p_0}\right)^{1/\gamma} = \left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(2(\gamma-1))} a_0 \frac{A}{V} t. \tag{7}$$

3 Subsonic flow

The assumption that the flow is choked breaks down when

$$p > \left(\frac{2}{\gamma+1}\right)^{\gamma/(\gamma-1)} p_0. \tag{8}$$

Let this value of the preessure be called p_c. When the pressure exceeds this value, the flow through the hole is subsonic and the mass flow rate is proportional to the pressure difference. The behavior of $p(t)$ during this time may be obtained approximately by determining the coefficient in the relation between flow rate and pressure difference by matching the value and rate of change of pressure at the point $p = p_c$. This leads to

$$\frac{dp}{dt} = C \frac{a_0 A}{V} (p_0 - p). \tag{9}$$

where

$$C = \gamma \frac{\left(\frac{2}{\gamma+1}\right)^{(3\gamma-1)/[2(\gamma-1)]}}{1 - \left(\frac{2}{\gamma+1}\right)^{\gamma/(\gamma-1)}}, \tag{10}$$

Integrating this from the point where $p = p_c$ and $t = t_c$, where t_c is the time at which the presure reaches p_c, we obtain

$$\frac{p}{p_0} = 1 - (1 - p_c/p_0) \exp[-C(t - t_c) a_0 A/V]. \tag{11}$$

The value of t_c is

$$t_c = \sqrt{\frac{\gamma+1}{2}} \frac{V}{a_0 A}. \tag{12}$$

A plot of a composite graph of equations (7) and (11) is shown in Figure 1. As may be seen, this suggests that the time it takes for the pressure sensor to read the ambient pressure is 5 times $V/(Aa_0)$.

The assumption that the flow rate is proportional to the pressure difference in this phase of the flow is not exactly correct, because compressibility is still a factor. Also, throughout the argument, viscous effects have been ignored. These will affect things like an effectively smaller value of A, and thus will increase the time delay. Nevertheless, the result in Figure 1 is likely to be a good rule for a broad range of conditions.

Figure 2 shows plots of these functions *vs.* γ.

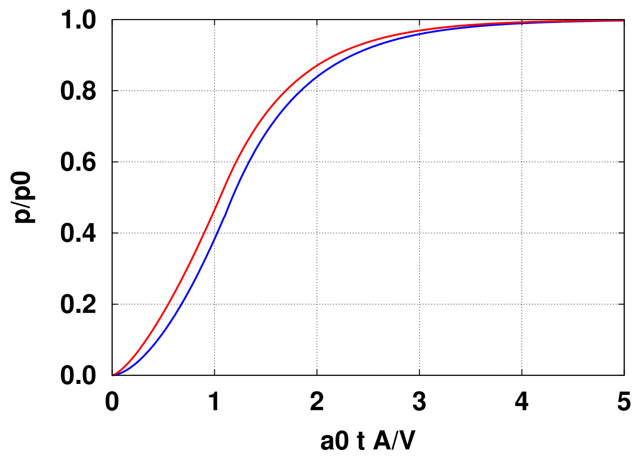

Figure 1: Composite graph of choked flow and subsonic flow for $\gamma = 1.4$ (red) and $\gamma = 5/3$ (blue).

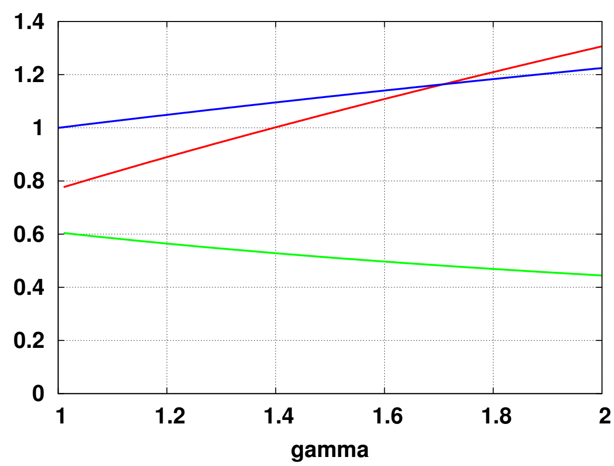

Figure 2: Plots of the constants $C/2$ (red), p_c/p_0 (green) and $t_c a_0 AV$ (blue) as functions of γ. As may be seen, the green, red and blue curves approach the limits $1/\sqrt{e}$ = 0.606, $1/(\sqrt{e} - 1)$ = 1.54, and 1 when $\gamma \to 1$, as they should.

4 Numerical computation

In order to check these theoretical results, a simple two–dimensional Euler computation was made. For the computations, the software system Amrita constructed by James Quirk was used. Amrita is a system that automates and packages computational tasks in such a way that the packages can be combined (dynamically linked) according to instructions written in a high–level scripting language. The present application uses features of Amrita that include the automatic construction of different Euler solvers, automatic adaptive mesh refinement according to simply chosen criteria, and scripting–language–driven computation, archiving and post–processing of the results.

The automation of the assembly and sequencing of the tasks makes for dramatically reduced possibility of hidden errors. More importantly, it makes computational investigations transparent and testable by others.

The ability to change one package at a time, without changing the rest of the scheme, permits easy detection of sources of error. The scope of the software system far exceeds its use here. The Euler solver generated for the present computations was an operator–split scheme with HLLE flux and kappa-MUSCL reconstruction. The computation was made with a Cartesian grid of 60×50 coarse–grid cells, and one tier of adaptive mesh refinement, by a factor of 4, so that the effective grid resolution was 240×200.

Figure 3 shows a comparison of the theoretical and numerical results, and it is clear that the disipative effects of weak shock waves bouncing around in the cavity cause the pressure to rise more slowly than with the isentropic flow assumption. It should be pointed out that the computation produces the correct asymptotic pressure only if a subsonic layer adjoins the surface. In order to produce a flow with a subsonic layer near the wall, the computations are performed with an imposed velocity distribution of the form

$$\frac{u}{u_\infty} = 1 - \exp(-y/\delta),$$

where y is the wall–normal distance, u is the wall–parallel velocity and u_∞ and δ are free stream speed and a parameter that measures the boundary layer thickness. This is imposed as initial condition and also as the inflow condition.

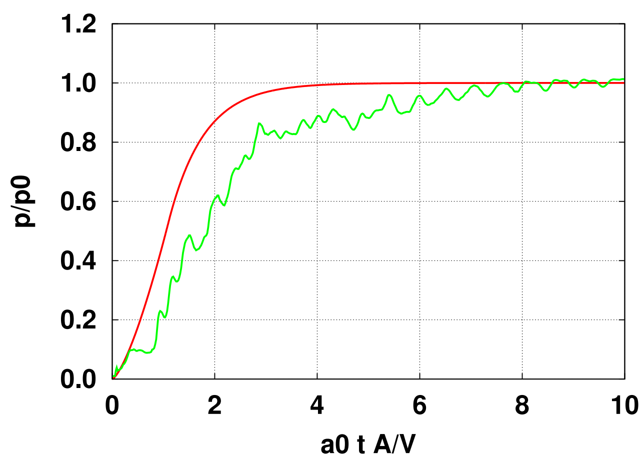

Figure 3: Comparison of theoretical and computational pressure traces for the case of $\gamma = 1.4$ and $M_\infty = 1.8$. A boundary layer velocity distribution is imposed, so that a subsonic layer adjoins the wall. The thickness of the subsonic layer is about the same as the hole width.