Impulse Correlation for Partially-Filled Detonation Tubes

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Introduction

The effect of nozzles on the impulse obtained from a detonation tube of circular cross section has been the focus of many experimental and numerical studies. In these cases, the simplified detonation tube is closed at one end (forming the thrust surface) and open at the other end enabling the attachment of an extension. A flow-field analysis in a detonation tube with an extension requires considering unsteady wave interactions making analytical and accurate numerical predictions difficult (especially in complicated extension geometries). In order to predict the impulse obtained from a detonation tube with an extension (considered a partially-filled detonation tube), we utilize data from other researchers to generate a partial-fill correlation.

Several experimental and numerical researchers have examined how the single-cycle impulse is affected by an extension. In these experimental studies, the tube is filled with the initial explosive mixture while the added extension is filled with an inert gas, usually atmospheric air. A thin diaphragm is used to separate the two mixtures. Zitoun and Desbordes[1] measured the impulse by integrating the thrust surface pressure differential of ethylene-oxygen mixtures at standard conditions in a tube with extensions having the same circular cross section. Zhdan et al.[2] directly measured the impulse using a ballistic pendulum arrangement of acetylene-oxygen mixtures at standard conditions in a tube with extensions

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having the same circular cross section. Similarly, Cooper et al.[3] and Falempin et al.[4] used a ballistic pendulum to measure impulse values of ethylene-oxygen mixtures in detonation tubes with attached extensions having a constant circular cross section and also in extensions of varying dimensions. Li and Kailasanath[5] numerically studied the effect of varying the length filled with the explosive mixture in tubes of constant cross sectional area on the impulse. They applied an exponential curve fit to their data relating the fuel-based specific impulse to the amount of the tube length filled with the explosive mixture.

These researchers either kept the tube length filled with the explosive mixture constant and added extensions of varying length or they kept the total tube plus extension length constant while varying the tube length filled with the combustible mixture. To obtain a quantitative measure of comparison between the different facilities, we calculate the fill fraction of the combined tube and extension assembly given the published dimensions. This work is to extend our previous modeling work on detonation tubes[6] to the case of partial filling and show how simple analytical estimates can be obtained for the impulse in both the fully and partially filled cases.

Data for partially-filled tubes

The data is plotted as a function of the fractional tube volume filled with the explosive mixture (Fig. 1). The single-cycle impulse I was normalized by the impulse I° for a fully-filled tube. The predictions of our single-cycle impulse model[6] for a fully-filled tube were used to normalize the experimental data of Zitoun and Desbordes[1] since experimental data of I° were not available.

Because a diaphragm of finite mass is used to separate the initial explosive mixture from the inert mixture of the extension in the experimental tests, incremental impulse is imparted to the tube due to the additional tamping mass of the diaphragm. For small tubes, even very thin diaphragms can equal a significant fraction of the initial explosive mixture mass, increasing the tamping effectiveness.[7] Based on information of the diaphragms provided by the researchers,[1,2,4,8] we use the Gurney model[7] to correct the measured impulse values for the diaphragm effect as described in Cooper and Shepherd.[8] Briefly, the Gurney model assumes a linear velocity gradient in the expanding product gases that are sandwiched between the driven mass (the detonation tube) and the tamper mass (the diaphragm). The final velocity of the driven mass is determined from the conservation equations and depends on the available chemical energy of the explosive. The impulse corrections applied, measured as a percentage of the measured impulse, are less than 2.3% for Cooper et al.,[3] less than 21.3% for Falempin et al.,[4] less than 22.3% for Zhdan et al.,[2] and less than 25.1% for Zitoun and Desbordes.[1] Figure 1 contains the corrected experimental data.

Partial-fill correlation

For the range of experimentally tested fill fractions ($0.15 < V/V^\circ < 1$), a linear relationship exists between the impulse fraction and the fill fraction

$$\frac{I}{I^\circ} = 0.814 \left(\frac{V}{V^\circ}\right) + 0.186 \ . \tag{1}$$

The numerical simulations by Li and Kailasanath[5] were used to determine the behavior of the partial-fill correlation at fill fractions close to zero ($V/V^\circ < 0.15$). They found that the impulse behavior near the origin in Fig. 1 can be approximated as

$$\frac{I}{I^\circ} = 3.560 \left(\frac{V}{V^\circ}\right) \ . \tag{2}$$

The intersection of these two linear relations, Eqs. (1) and (2), occurs at a fill fraction of 0.0676 determining the range of applicability for each equation.

As shown in Fig. 1, the maximum impulse from a detonation tube is obtained by completely filling it with the explosive mixture. In other words, filling only a fraction of the tube volume with the explosive mixture results in obtaining only a fraction of the maximum possible impulse. Equations (1) and (2), written in terms of impulse, can be rewritten as mixture specific impulse $I_{sp} = I/g\rho_1 V$ normalized by the specific impulse I°_{sp} of the fully-filled tube. The initial explosive mixture density is represented by ρ_1 and g is the standard gravitational acceleration. For $0.0676 < V/V^\circ < 1$

$$\frac{I_{sp}}{I^\circ_{sp}} = 0.814 + 0.186 \left(\frac{V^\circ}{V}\right) \ , \tag{3}$$

and for $0 < V/V^\circ < \ 0.0676$

$$\frac{I_{sp}}{I^\circ_{sp}} = 3.560 \ . \tag{4}$$

The data of Fig. 1 are replotted in terms of specific impulse in Fig. 2. The specific impulse is found to increase as the explosive mixture mass decreases indicating a specific performance increase even though the total impulse decreases. In the limit as the explosive mass tends to zero, the specific impulse ratio tends to a constant value.

The crucial issue in understanding how the impulse behaves as a function of the fill fraction is to consider the relative masses of the detonation tube, combustible mixture, and the inert gases. Because the combustible mixture has a constant amount of stored chemical energy per unit mass, it is the distribution of this chemical energy into accelerating the tube, product gases, and inert gases that determine the impulse imparted to the tube.

Considering energy and momentum conservation, the impulse imparted to the tube must equal the impulse imparted to the expanding detonation products and the inert gases. Thus, for a constant combustible mixture mass, increasing the mass of inert gas decreases the exit velocity of the gases. This results in more of the stored chemical energy to be imparted to the tube causing an increase in the specific impulse as shown in Fig. 2. In the limit of infinite inert gas mass (or zero combustible mixture mass), the average gas exit velocity goes to zero and the maximum amount of stored chemical energy goes into driving the tube. This situation corresponds to a tube of infinite length where the specific impulse reaches a limiting value.

The impulse curve of Fig. 1 is based on the total mass of the combustible mixture and the inert gases contained in the tube. The mass of the combustible mixture and inert gases depend on their initial density and fill fraction. For the experimental data discussed above, the densities of the combustible mixture (ethylene-oxygen or acetylene-oxygen) and inert gas (atmospheric air) are approximately equal. This means that the total mass within a tube of constant length remains approximately constant regardless of fill fraction. By decreasing the amount of combustible mixture in the tube, a corresponding decrease in the available stored energy within the tube occurs. As a result, decreasing the fill fraction decreases the impulse imparted to the tube such that a fully-filled tube produces the maximum impulse since the available stored chemical energy is maximized, while a tube containing only inert gases (V/V^0=0) produces zero impulse since the available chemical energy equals zero.

To summarize, our partial-fill correlation consists of the two relationships, Eqs. (1) and (2) for impulse or alternatively, Eqs. (3) and (4) for specific impulse. This correlation is empirical in nature and is derived from a limited amount of experimental and numerical data. However, as shown subsequently, it compares very well with multi-cycle data over a wide range of fill fractions. Its advantages are that it is simple and in conjunction with our previous models of fully-filled tubes, provides a rapid means of estimating the ideal impulse of partially-filled detonation tubes.

Comparisons with partial-fill correlation

Our partial-fill correlation, Eq. (3) for $0.0676 < V/V^\circ < 1$ and Eq. (4) for $0 < V/V^\circ < 0.0676$, is compared to multi-cycle experiments by Schauer et al.[9] in hydrogen-air mixtures (Fig. 3). Data were obtained for a variety of tube dimensions, fill fractions, and cycle frequencies. Impulse and thrust measurements were taken with a damped thrust stand and, for our correlation, we assume that multi-cycle operation is equivalent to a series of ideal single cycles. This data were not considered in the development of the partial fill correlation enabling an independent test to experimental data for validation purposes.

The fill fractions in Fig. 3 greater than one correspond to over-filling the detonation tube, and in this case, the *specific* impulse is reduced since only the mixture within the tube contributes to the impulse. The impulse I of an over-filled tube is equal to the impulse I° of a fully-filled tube. This can be simply accounted for by computing the specific impulse as

$$\frac{I_{sp}}{I_{sp}^\circ} = \left(\frac{I}{\rho_1 g V}\right)\left(\frac{\rho_1 g V^\circ}{I^\circ}\right) = \frac{V^\circ}{V} \tag{5}$$

when $V/V^\circ > 1$. This relation is precise and valid for all fill fractions greater than one.

Li and Kailasanath[5] proposed a correlation for specific impulse of partially-filled tubes based on an exponential curve fit with data from their numerical simulations

$$\frac{I_{spf}}{I_{spf}^\circ} = a - \frac{(a-1)}{exp\left(\dfrac{L^\circ/L - 1}{8}\right)} \ . \tag{6}$$

The constant a has values[5] between 3.2 and 3.5.

Equation (6) in terms of fill fractions are compared with our partial-fill correlation (Fig. 1). Both relationships predict zero impulse at a fill fraction of zero as expected. Our partial-fill correlation and the curve fit from the numerical simulations both tend to a constant specific impulse value in the limit of zero explosive mixture.

Conclusions

A simple correlation has been developed to predict the impulse in partially-filled detonation tubes. The correlation was based on interpretation of published experimental[1–4] and numerical[5] data. A piecewise linear correlation is found to adequately describe the existing single-cycle and multi-cycle data for a wide range of fill fractions. The impulse increases with increasing fill fraction and the maximum value is obtained in a full tube. The specific impulse increases with decreasing fill fraction and the maximum value is obtained in the limit of vanishing explosive mixture amount. The Gurney model was utilized to correct the experimental data for the impulse increment that is a result of a finite diaphragm mass.

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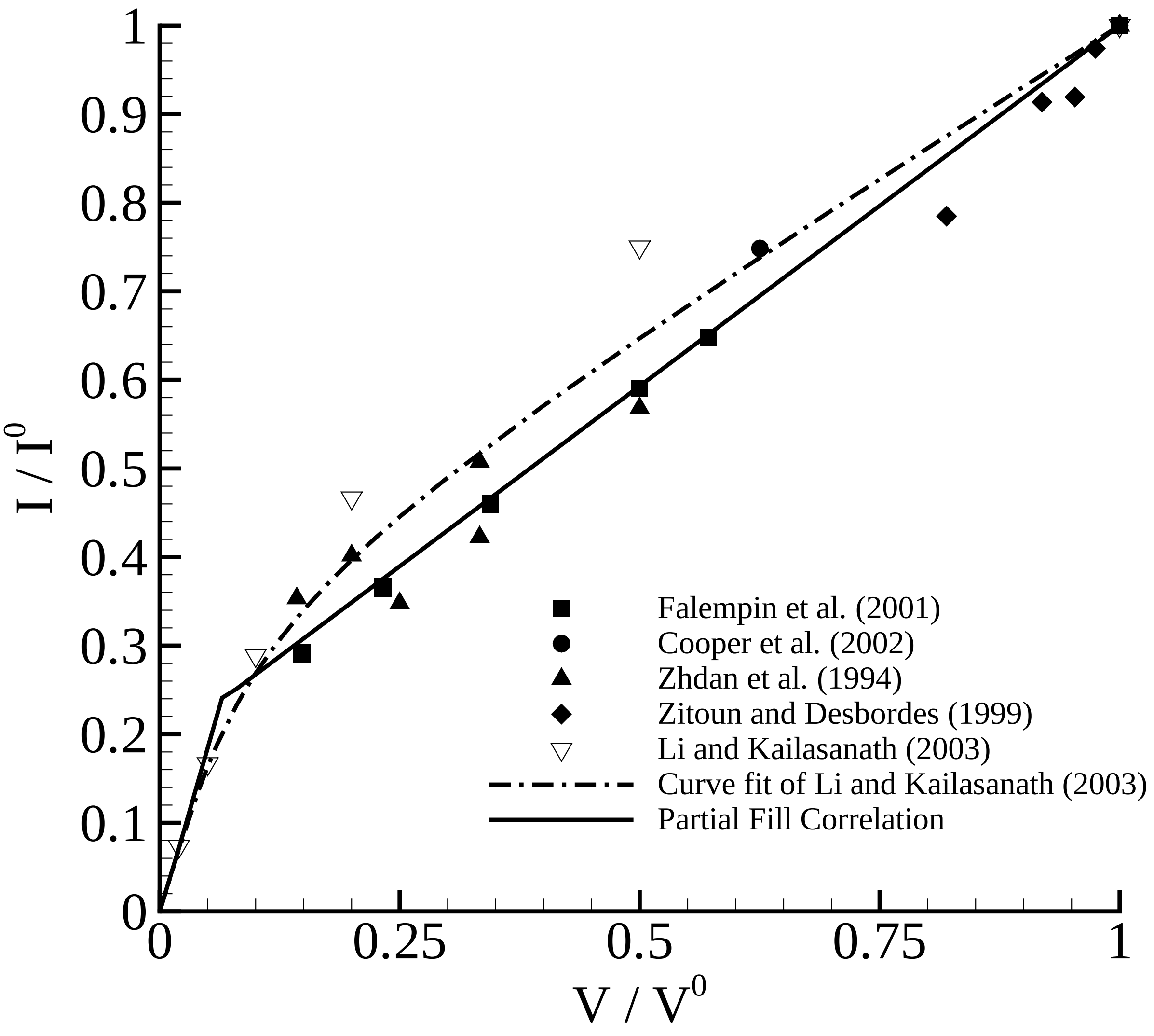

Figure 1: **Cooper et al.**

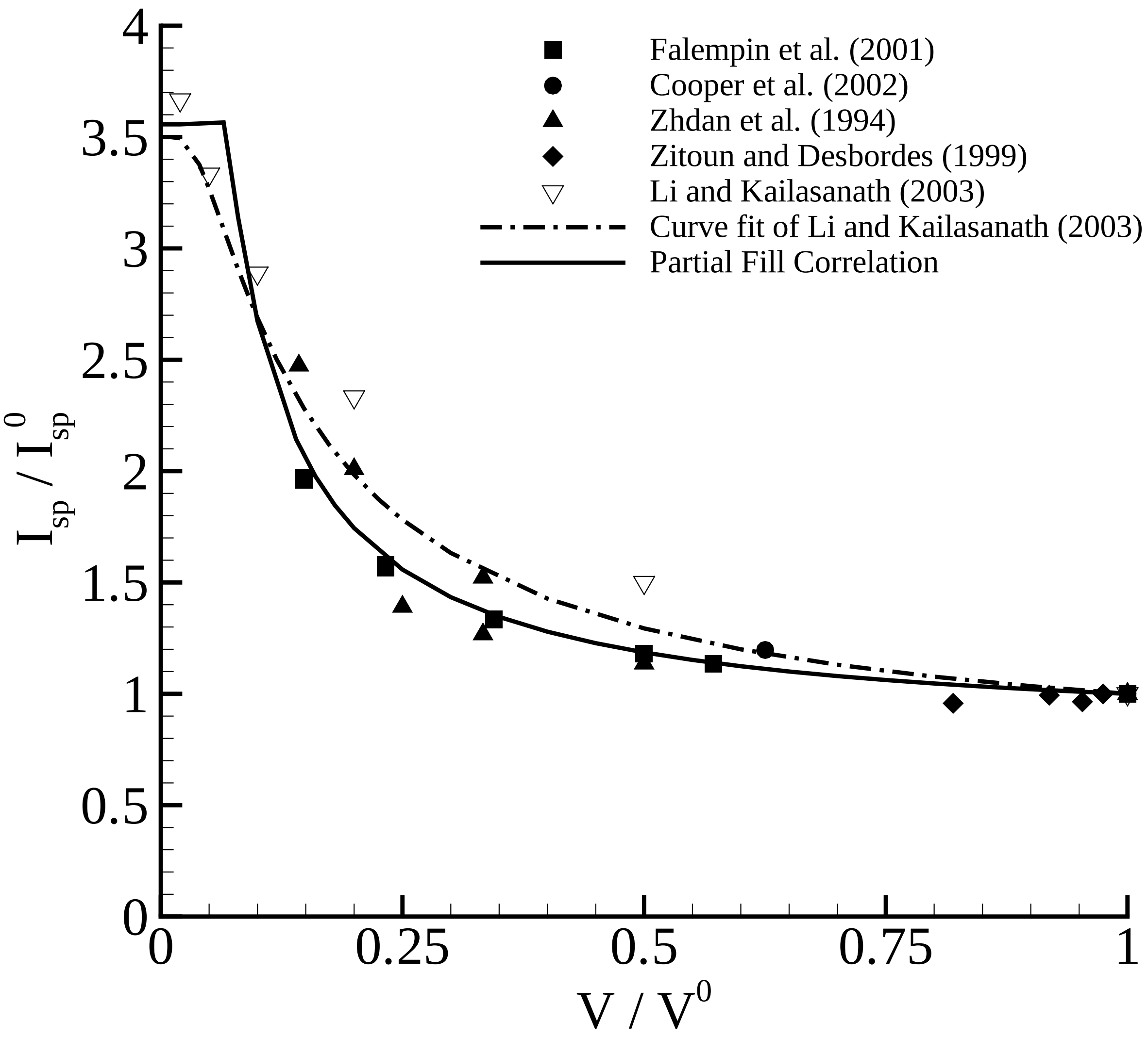

Figure 2: Cooper et al.

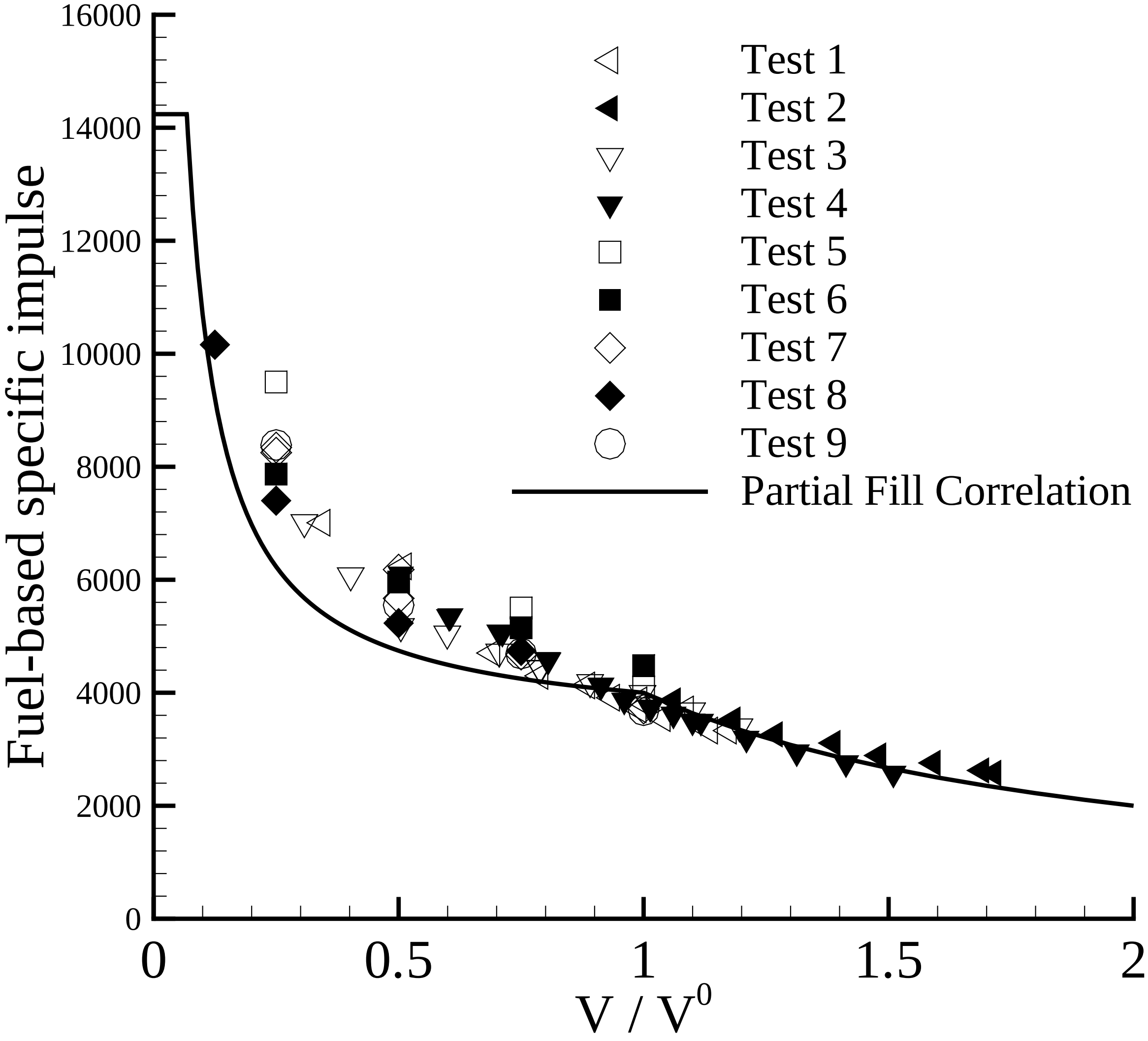

Figure 3: **Cooper et al.**