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The effect of nozzles on the impulse obtained from a detonation tube has been the focus of many experimental and numerical studies. We develop a partial-fill model to predict the impulse obtained from a detonation tube containing an extension (considered a partially-filled detonation tube). The experimental impulse values are found to be linearly dependent on the fraction of the tube volume filled with the explosive mixture. Data from numerical simulations were used to predict the impulse for small fill fractions not experimentally tested. The analytical Gurney model provides a method for correcting the experimental data for the tamping provided by diaphragms of a finite mass. A thermodynamic cycle analysis of a detonation is conducted to evaluate the fraction of stored chemical energy in an explosive mixture that can be converted into mechanical work. It is found that approximately 46%-64% of the ideal work from a detonation can be converted into impulse. This fraction increases with increasing nitrogn dilution. The partial-fill model is validated with multi-cycle experimental data and numerical simulations. We compare the partial-fill model with experimental data of diverging nozzles.

Nomenclature

C	explosive mixture mass
E	Gurney energy
E_{Ideal}	ideal energy calculated from Jacobs cycle
E_{Isp}	energy based on predicted specific impulse
g	standard gravitational acceleration
h_1	specific enthalpy of reactants
h_4	specific enthalpy of products at state 4 of Jacobs cycle
H_C	heat of combustion
I	impulse of partially-filled tube
I°	impulse of tube fully filled with combustible mixture
I_{sp}	mixture-based specific impulse of partially-filled tube
I°_{sp}	mixture-based specific impulse for fully-filled tube
I_{spf}	fuel-based specific impulse of partially-filled tube
I°_{spf}	fuel-based specific impulse for fully-filled tube
L	tube length filled with explosive mixture
L°	total length of tube and extension
M	detonation tube mass
N	tamper mass
P_1	pressure of reactants
P_2	Chapman-Jouguet pressure
P_3	pressure of burned gases behind Taylor wave
t	time
t_1	time required by detonation wave to travel tube length filled with explosive mixture
T_1	temperature of reactants
T_2	Chapman-Jouguet temperature
u_1	specific internal energy of reactants
u_3	specific internal energy of products at state 3 of Jacobs cycle
U_{CJ}	Chapman-Jouguet detonation velocity
x	distance along detonation tube
V	tube volume filled with explosive charge
V°	total volume of tube and extension
ϵ_{Ideal}	ideal detonation efficiency calculated from Jacobs cycle
ϵ_{Isp}	detonation efficiency based on predicted specific impulse
ρ_1	density of reactants
ρ_{air}	density of air mixture in extension

Introduction

IN an effort to maximize the impulse delivered by an explosive mixture for pulse detonation engine (PDE) applications, researchers have begun studying the effect of various tube exit conditions. A simplified detonation tube consists of a cylindrical tube closed at one end (forming the thrust surface) and open at the other end enabling the attachment of a nozzle. The nozzle can be of any type such as converging, diverging, converging-diverging, cylindrical, and bell shaped to name just a few examples. We categorize these different nozzle shapes as extensions and the tube's exit condition is said to be modified if an extension is attached to its open end. This paper utilizes data from other researchers to develop a model of how the impulse is affected by modifications to the tube's exit condition through the use of extensions.

Numerous experimental and numerical studies have examined how the single-cycle impulse is affected by an extension. In these cases, the tube is filled with the initial explosive mixture while the added extension is filled with an inert gas, usually atmospheric air. A thin diaphragm is used to separate the two mixtures. Zitoun and Desbordes[1] measured the single-cycle impulse of ethylene-oxygen mixtures at standard conditions in a tube and extension with the same cylindrical cross section. They directly initiated a detonation with approximately 35 J of energy and the impulse was calculated by integrating the thrust surface pressure differential. Zhdan et al.[2] directly measured the single-cycle impulse of acetylene-oxygen mixtures at standard conditions also in a tube and extension with the same cylindrical cross section. They used a ballistic pendulum to measure the impulse from the tube's maximum horizontal displacement. Cooper et al.[3] and Falempin et al.[4] also used a ballistic pendulum to measure single-cycle impulse values of ethylene-oxygen mixtures in detonation tubes with attached extensions having a constant cylindrical cross section and also in extensions of varying dimensions. Cooper et al.[3] extended their tests to study the effect of diluent amount.

Several numerical studies of extensions have also been completed. Yang et al.[5] calculated the impulse for a converging, diverging, and plug nozzle in hydrogen-air mixtures at 0.29 atm and 228 K. Li and Kailasanath[6] studied the effect of varying the length filled with the explosive mixture in tubes of constant cross-sectional area. They applied an exponential curve fit to their data relating the fuel-based specific impulse to the amount of the tube length filled with the explosive mixture. Cambier and Tegner[7] studied the effect of diverging nozzle exit area on impulse in hydrogen-oxygen mixtures at 1 atm and 350 K. Eidelman et al.[8] studied the effect of several converging, diverging, and straight extensions on impulse in acetylene-air at standard conditions.

These researchers either kept the tube length filled with the explosive mixture constant and added extensions of varying length or they kept the total tube plus extension length constant while varying the tube length filled with the mixture. Thus, a detonation tube with its extension can be described in two ways. The most straightforward description consists of a detonation tube with a separate extension attached to the tube's open end. In this description, the detonation tube is assumed to have a constant cross-sectional area and is filled with the initial explosive mixture. The attached extension may be of varying cross-sectional area and is filled with an inert mixture, typically atmospheric air. The second description consists of referring to the *combined* tube and extension assembly as the detonation tube. In this case, the portion of the tube containing the explosive mixture is of constant cross-sectional area while the remaining portion of the tube containing the inert mixture may be of varying cross-sectional area. Because the tube and extension are considered together, the volume fraction of the tube filled with the initial explosive mixture can be used as a quantitative measure of comparison between different facilities. We use this approach for the remainder of the paper, calculating the fill fraction of the combined tube and extension assembly given the published dimensions of the attached extension.

The analysis of the flow field in a detonation tube with an extension requires considering unsteady wave interactions. Analytical and accurate numerical predictions (especially in complicated extension geometries) prove difficult, suggesting the use of experimental data in developing a correlating relation between an extension and its associated impulse. We begin with a discussion of the gas dynamics inside detonation tubes that are fully and partially filled with an initial explosive mixture. A model of partial filling is generated for the experimental data referenced above in tubes of constant cylindrical cross section. This paper does not consider the effect of converging extensions on impulse because the increased blowdown times characteristic of the exit restrictions dominate the effect on impulse and should be addressed in a separate paper.

Detonation tube gas dynamics

Three cases of interest exist for partially-filled detonation tubes of a simple geometry (Fig. 1). Case A consists of a finite length tube completely filled with the explosive mixture, case B consists of a finite length tube partially filled with the explosive mixture, and case C consists of an infinitely long tube with a section filled with explosive mixture. Case C may be thought of as the limit of case B as the explosive mixture mass tends to zero (or the mass of the inert gas tends to infinity). To discuss the flow field of the three cases we consider an ideal detonation directly initiated at the tube's closed end. The detonation propagates at the Chapman-Jouguet velocity towards the tube's open end.

Case A, consisting of a finite length tube completely filled with the combustible mixture, has been studied extensively, both experimentally[1–4] and numerically.[5–8] The gas dynamic processes have been modeled[9] generating estimates of the thrust wall pressure history and impulse for a variety of initial mixtures (Eq. 1).

$$I^\circ = 4.3V^\circ \frac{(P_3 - P_1)}{U_{CJ}} \tag{1}$$

As shown in the distance-time diagram of Fig. 2, the detonation reaches the open end of the tube resulting

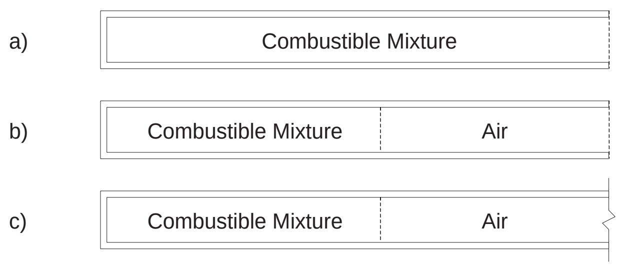

Fig. 1 Detonation tube cases: A) Finite length tube entirely filled with combustible mixture. B) Finite length tube partially filled with combustible mixture. C) Infinite length tube with a section filled with combustible mixture.

in a shock being transmitted outside the tube. A reflected wave, usually an expansion, propagates back to the thrust surface. After the reflected wave reaches the thrust surface, the pressure inside the detonation tube begins to decrease, eventually matching the environment pressure.

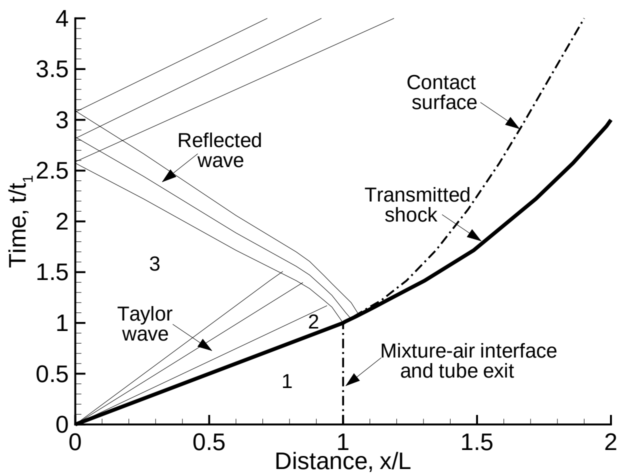

Fig. 2 Distance-time diagram for finite length tube of case A fully filled with explosive mixture.

When the tube is partially filled as in case B, the detonation wave ceases when it reaches the explosive mixture-air interface (Fig. 3). A reflected wave propagates back to the thrust surface and a transmitted wave travels through the air-filled extension. When this transmitted wave reaches the area change at the tube exit, a second wave is reflected back toward the thrust surface. There are many subsequent wave interactions and the simple one-dimensional analysis[9] used sucessfully in case A is difficult to apply and analytical estimates of the impulse do not appear to be possible.

For case C, the first reflected wave at the mixture-air interface propagates back to the thrust wall and the transmitted wave travels through the extension (Fig. 4). Since the tube is infinitely long, a second wave reflection at the tube's open end does not occur and the thrust surface pressure remains higher for a period of time longer than in cases A or B.

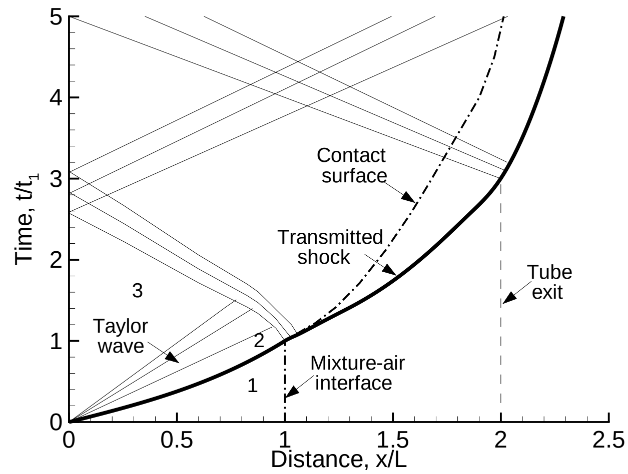

Fig. 3 Distance-time diagram for finite length tube of case B partially filled with explosive mixture.

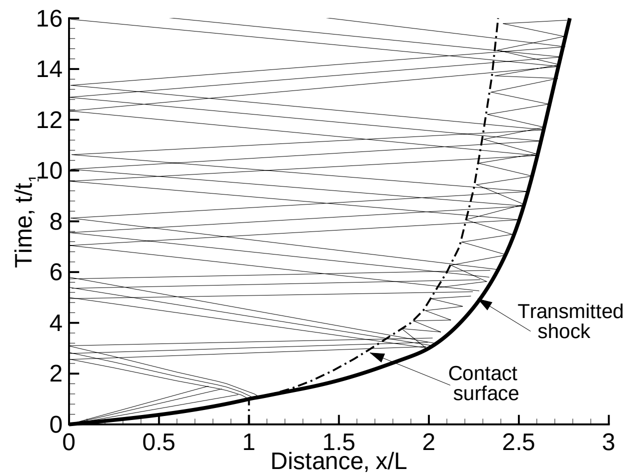

Fig. 4 Distance-time diagram for infinite length tube of case C with a portion filled with explosive mixture.

In general, because of the numerous wave interactions and the complex flow field resulting from the area change at the tube exit, simple one-dimensional gas dynamics analysis predicting the impulse is not possible. Instead, experimental and numerical data from multi-dimensional simulations are used to generate a model for impulse that depends on the fraction of the tube volume filled with explosive mixture.

Data for partially-filled tubes

The experimental data referenced in the introduction is collected and plotted as a fraction of the tube volume filled with the explosive mixture (Fig. 5). The single-cycle impulse I was normalized by the impulse I° for a fully-filled tube. In cases where experimental data were not available, model[9] predictions of I° were used.

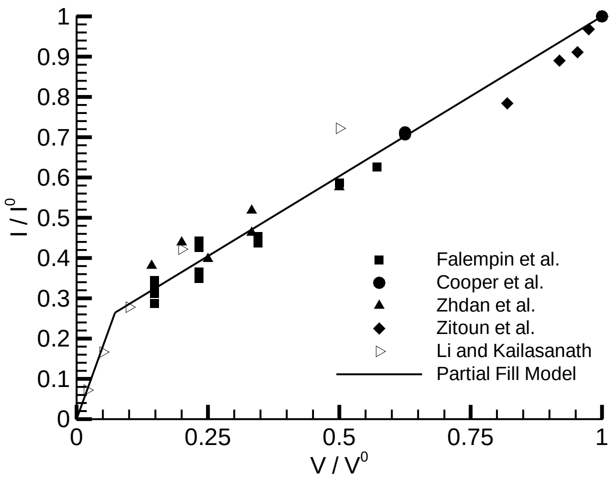

Fig. 5 Normalized impulse from published data for tubes with constant cylindrical cross section and comparison to partial-fill model.

Partial-fill model

For the range of experimentally tested fill fractions ($0.15 < V/V^\circ < 1$), a linear relationship exists between the impulse fraction and the fill fraction

$$\frac{I}{I^\circ} = 0.794\left(\frac{V}{V^\circ}\right) + 0.206\ . \quad (2)$$

It is clear that at zero fill fraction, the impulse should go to zero for case B and the specific impulse for case C will reach a limiting value. This indicates that the impulse will be linearly proportional to the fill fraction for very small values of V/V°. The numerical simulations by Li and Kailasanath[6] were used to determine the partial-fill model behavior at fill fractions close to zero. They found that the specific impulse I_{sp} for very small fill fractions simulating case C was about 3.6 times the value I_{sp}° for case A. This suggests that the behavior near the origin in Fig. 5 can be approximated as

$$\frac{I}{I^\circ} = 3.6\left(\frac{V}{V^\circ}\right)\ . \quad (3)$$

The intersection of these two linear relations, Eqs. (2) and (3), occurs at a fill fraction of 0.073, determining the range of applicability for each equation.

As shown in Fig. 5, the maximum impulse from a detonation tube is obtained by completely filling it with the explosive mixture. In other words, filling only a fraction of the tube volume with the explosive mixture results in obtaining only a fraction of the maximum possible impulse. Equations (2) and (3), written in terms of impulse, can be rewritten as mixture specific impulse $I_{sp} = I/g\rho_1 V$ normalized by the specific impulse I_{sp}° of the full tube. For $0.073 < V/V^\circ < 1$

$$\frac{I_{sp}}{I_{sp}^\circ} = 0.794 + 0.206\left(\frac{V^\circ}{V}\right)\ , \quad (4)$$

and for $0 < V/V^\circ < 0.073$

$$\frac{I_{sp}}{I_{sp}^\circ} = 3.6\ . \quad (5)$$

The data of Fig. 5 are replotted in terms of specific impulse in Fig. 6. The specific impulse is found to increase as the explosive mixture mass decreases indicating a specific performance increase even though the impulse decreases. In the limit as the explosive mass tends to zero (case C), the specific impulse ratio tends to a constant value.

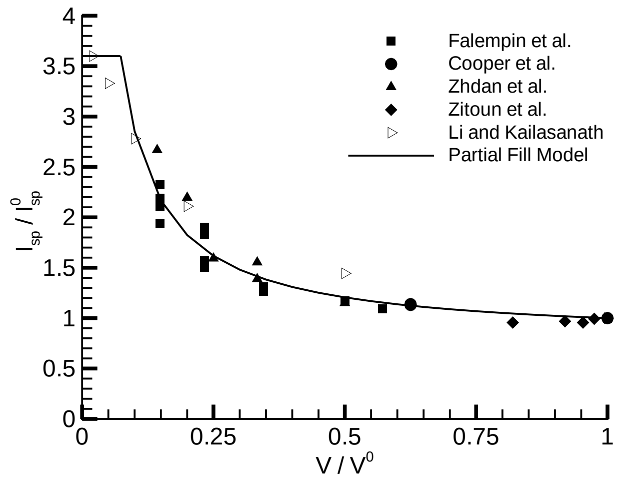

Fig. 6 Partial-fill model in terms of specific impulse with published data for tubes with constant cylindrical cross section.

To summarize, our partial-fill model consists of the two relationships, Eqs. (2) and (3) for impulse or alternatively, Eqs. (4) and (5) for specific impulse. This model is empirical in nature and is derived from a limited amount of experimental and numerical data. However, as shown subsequently, it compares very well with multi-cycle data over a wide range of fill fractions. Its advantages are that it is simple and in conjunction with our previous models of fully-filled tubes, provides a rapid means of estimating the ideal impulse of partially-filled detonation tubes.

The Gurney Model

A model developed by R. W. Gurney for predicting explosive fragment velocities can be applied to the PDE situation. This model suggests a simple approach to the prediction of impulse, provides some physical insights into the partial-fill case, and can be used to correct measured impulses for the effect of diaphragm mass. The Gurney model[10] predicts the velocity and impulse of a body with mass M driven by the detonation of an explosive charge with mass C. It can be extended to situations where the explosive is also confined by a tamper mass N. In case of a detonation tube, M is the mass of the detonation tube, C is the mass of the explosive mixture, and N is the mass of the inert gas filling the extension plus the mass of the diaphragm. The typical situation for a detonation tube is that the mass of the tube is large compared to the mass of the charge, $M/C \rightarrow \infty$.

The Gurney model considers the mass M and the tamper N to be rigid bodies and assumes a linear velocity distribution in the explosion products. Using the conservation of mass, momentum, and energy, the ultimate velocities of the masses M and N can be predicted when the gas has expanded to ambient pressure. In the case of a large mass-to-charge ratio M/C, it is the impulse on the mass M that is predicted. The Gurney model assumes one-dimensional motion of the propelled body and tamper mass in addition to an assumed linear velocity profile of the product gases. For a detonation tube, these assumptions are not strictly satisfied but, nevertheless, useful ideas can be obtained from this model.

Using the Gurney model, the explosive mixture-based specific impulse can be written as a function[10] of the body mass, explosive mixture mass, tamper mass, and Gurney energy E for a one-dimensional slab geometry of explosive sandwiched between the tamper and the body.

$$I_{sp} = \frac{\sqrt{2E}}{g} \frac{\left(\frac{N}{C} + \frac{1}{2}\right)}{\sqrt{\left(\frac{N}{C} + \frac{1}{3}\right)}} \tag{6}$$

For a fully-filled tube without any tamping $N = 0$, this can be written as

$$I^{\circ}_{sp} = \frac{\sqrt{1.5E}}{g} \; . \tag{7}$$

The tamper mass can be related to the fraction of the tube volume filled with the inert mixture by

$$\frac{V^{\circ}}{V} = \frac{\rho_1}{\rho_{air}} \frac{N}{C} + 1 \; . \tag{8}$$

For a given explosive mixture, the specific impulse then depends on the fill fraction and the energy E representing the energy stored in the explosive mixture available to do mechanical work. Equation (6) can be solved directly to determine the impulse as a result of the tamping provided by a finite diaphragm mass.

Diaphragm mass correction

Because a diaphragm of finite mass is used to separate the initial explosive mixture from the inert mixture in the experimental tests, an impulse increase results. This effect can be accounted for by determining the incremental impulse imparted to the tube due to the additional tamping mass provided by the diaphragm. For small tubes, even very thin diaphragms can equal a significant fraction of the initial explosive mixture mass increasing the tamping effectiveness.[10] Based on information provided by the researchers,[1–4] the ratio of the diaphragm mass to explosive mixture mass for each fill fraction was calculated. The impulse from the diaphragm tamping is calculated with Eq. (6). The impulse from the same explosive mass without tamping is calculated with Eq. (7) and the two results subtracted to determine the contribution of the diaphragm mass on the impulse. The measured impulse values were then corrected for the diaphragm effect by subtracting the additional impulse due to the diaphragm tamping. Fig. 5 contains the corrected experimental data.

Gurney Energy and Maximum Work by Detonations

The Gurney model assumes that every explosive can be characterized by a specific energy E that determines the amount of mechanical work (acceleration of the surrounding metal) that can be done by the explosive. The Gurney energy E is only a fraction of the stored chemical energy in the initial explosive mixture since the explosion products are still hot when expanded to ambient pressure. The value of the parameter E is needed in order to use the Gurney model to predict impulse. For high explosives, this value is well known from carefully conducted experiments and can be easily approximated with simple relations that depend on the explosive's detonation parameters.[10] However, these relations are not applicable for the gaseous mixtures used in PDE situations. To further complicate matters, the energy available for mechanical work is often sensitive to variations in initial density, temperature, and degree of confinement.[11] For this reason, we have carried out a series of computations to evaluate E for gaseous mixtures of interest to PDE applications and compared the values to both the total available chemical energy and the ideal amount of work that can be obtained from a detonation process.

The total amount of work obtained from combusting an explosive mixture can be calculated by considering a series of processes or cycle connecting a sequence of equilibrium states. When a series of ideal processes is considered, the maximum possible work can be calculated. This is the basis of traditional thermodynamic cycle analysis; for example, the analysis of steady gas turbine engines is based on the Brayton cycle. Although a detonation is unsteady and irreversible, a cycle developed by Jacobs in 1956 can be used to calculate the maximum work available from the detonation of an explosive mixture.[12]

The equilibrium states and intermediate processes of the Jacobs cycle for several explosive mixtures (Fig. 7) were calculated with realistic thermodynamics using STANJAN.[13] Reactants at the initial conditions (state 1) are processed by a detonation wave to from products moving with a uniform velocity at the Chapman-Jouguet conditions (state 2). This process is denoted by the Rayleigh line connecting states 1 and 2. Mechanical work is extracted from the kinetic energy of the detonation products at state 2 and then the products are isentropically expanded through a series of equilibrium states to the initial pressure (state 3). Heat is then removed from the products at constant pressure until the initial reactant temperature is reached (state 4). This sequence of processes accounts for both the internal thermodynamic and kinetic energies of the detonation products. The final process to convert the products at state 4 into the reactants at state 1 requires heat addition at constant pressure and temperature. The value of this heat addition is equal to the heat of combustion defined by

$$H_C = h_1 - h_4 \tag{9}$$

assuming water vapor in the products at state 4 (Table 1).

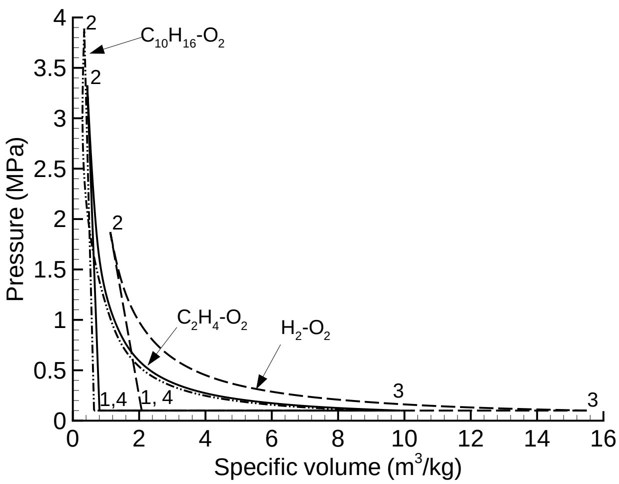

Fig. 7 Jacobs thermodynamic cycle for $C_{10}H_{16}$-, C_2H_4-, and H_2-O_2 mixtures at initial conditions of 100 kPa and 300 K.

In most explosive applications, including PDEs, the heat transfer process between states 3 and 1 is an irreversible loss of heat to the surroundings and is not converted into mechanical work.[12] The maximum possible mechanical work from a detonation process therefore considers only the net work from processes between states 1 and 3. Using elementary thermodynamics and the conservation relations for an ideal detonation, the amount of energy E_{Ideal} available to do mechanical work is

$$E_{Ideal} = u_1 - u_3 \ . \tag{10}$$

This can be represented on the pressure-volume diagram of Figure 7 as the area under the processes 1-2-3. The values for several explosive mixtures are tabulated in Table 1 as a function of the mixture mass.

For mixtures with a higher temperature at the end of the expansion process, more of the useful work is lost through irreversible heat transfer during the cooling processes between states 3 and 1 implying that mixtures with a higher heat of combustion may not necessarily provide more mechanical work.

Mixture	E_{Ideal} [MJ/kg]	H_C [MJ/kg]	ϵ_{Ideal}
H_2-O_2	3.90	13.29	0.294
H_2-O_2-20%N_2	2.90	8.39	0.345
H_2-O_2-40%N_2	2.14	5.20	0.411
H_2-Air	1.55	3.39	0.458
C_2H_2-O_2	3.19	11.82	0.270
C_2H_2-O_2-20%N_2	2.81	9.60	0.292
C_2H_2-O_2-40%N_2	2.41	7.31	0.330
C_2H_2-O_2-60%N_2	1.95	4.95	0.394
C_2H_2-Air	1.54	3.39	0.454
C_2H_4-O_2	3.13	10.67	0.293
C_2H_4-O_2-20%N_2	2.75	8.70	0.316
C_2H_4-O_2-40%N_2	2.36	6.66	0.354
C_2H_4-O_2-60%N_2	1.89	4.53	0.416
C_2H_4-Air	1.41	3.01	0.469
C_3H_8-O_2	3.17	10.04	0.316
C_3H_8-O_2-20%N_2	2.80	8.33	0.336
C_3H_8-O_2-40%N_2	2.40	6.48	0.371
C_3H_8-O_2-60%N_2	1.91	4.49	0.426
C_3H_8-Air	1.34	2.80	0.478
$C_{10}H_{16}$-O_2	3.04	9.83	0.309
$C_{10}H_{16}$-O_2-20%N_2	2.72	8.34	0.327
$C_{10}H_{16}$-O_2-40%N_2	2.38	6.65	0.358
$C_{10}H_{16}$-O_2-60%N_2	1.95	4.73	0.411
$C_{10}H_{16}$-Air	1.33	2.79	0.476

Table 1 Ideal work and efficiency values calculated with the Jacobs cycle for several explosive mixtures at initial conditions of 100 kPa and 300 K.

Table 1 contains values of the ideal explosive efficiency

$$\epsilon_{Ideal} = E_{Ideal}/H_C \tag{11}$$

which represents the fraction of the stored chemical energy in the explosive mixture converted into mechanical work assuming ideal processes. In the case of a detonation tube, the hot products must fully expand both adiabatically and reversibly to the initial reactant pressure inside the tube for the thrust surface pressure differential to follow the isentrope between states 2 and 3. In reality, the detonation products exhaust from the tube's open end at pressures significantly higher than the initial reactant pressure resulting in incomplete expansion within the tube. Product gas expansion occurring outside the tube does not contribute to the thrust and results in work lost to the environment.

Considering a detonation tube completely filled with explosive mixture (case A), we can use the Gurney model to determine the fraction of the ideal work that goes into generating impulse. Using the expression for the untamped specific impulse, Eq. (7), we define an energy E_{Isp} by

$$E_{Isp} = \frac{(gI^{\circ}_{sp})^2}{1.5} \ . \tag{12}$$

Specific impulse values for a fully-filled tube predicted by Wintenberger et al.[9] are used to evaluate the energy and corresponding impulse generation efficiency

$$\epsilon_{Isp} = \frac{E_{Isp}}{H_C} \tag{13}$$

for several mixtures (Table 2). The specific impulse can now be predicted based on thermodynamic considerations

$$I^{\circ}_{sp} = \frac{\sqrt{1.5\epsilon_{Isp}H_C}}{g} . \tag{14}$$

Mixture	I_{sp} [s]	E_{Isp} [MJ/kg]	ϵ_{Isp}
H_2-O_2	172.9	1.92	0.144
H_2-O_2-20%N_2	155.4	1.55	0.185
H_2-O_2-40%N_2	138.7	1.23	0.237
H_2-Air	123.7	0.98	0.290
C_2H_2-O_2	150.9	1.46	0.124
C_2H_2-O_2-20%N_2	146.0	1.37	0.143
C_2H_2-O_2-40%N_2	139.8	1.25	0.171
C_2H_2-O_2-60%N_2	130.6	1.09	0.221
C_2H_2-Air	120.6	0.93	0.275
C_2H_4-O_2	151.0	1.46	0.137
C_2H_4-O_2-20%N_2	145.7	1.36	0.156
C_2H_4-O_2-40%N_2	139.1	1.24	0.186
C_2H_4-O_2-60%N_2	129.3	1.07	0.237
C_2H_4-Air	117.0	0.88	0.292
C_3H_8-O_2	152.7	1.50	0.149
C_3H_8-O_2-20%N_2	147.3	1.39	0.167
C_3H_8-O_2-40%N_2	140.4	1.26	0.195
C_3H_8-O_2-60%N_2	130.3	1.09	0.243
C_3H_8-Air	115.4	0.85	0.305
$C_{10}H_{16}$-O_2	148.4	1.41	0.144
$C_{10}H_{16}$-O_2-20%N_2	144.1	1.33	0.160
$C_{10}H_{16}$-O_2-40%N_2	138.5	1.23	0.185
$C_{10}H_{16}$-O_2-60%N_2	130.1	1.09	0.229
$C_{10}H_{16}$-Air	114.6	0.84	0.302

Table 2 Energy and efficiency values based on predicted[9] specific impulse values for several mixtures at initial conditions of 100 kPa and 300 K.

As shown in Table 2, the efficiency values range between 0.124 and 0.305 for the gaseous fuel-oxygen-nitrogen mixtures. These values are slightly less than typical propellant efficiency values of 0.2-0.3 and are significantly less than typical efficiency values of 0.6-0.7 for high explosives.[10]

This study of the thermodynamic cycle for a detonation has generated values for the maximum mechanical work available from the chemical energy stored in an explosive mixture. We have used the untamped Gurney model equation and predicted[9] specific impulse values to calculate a realistic estimate for the amount of work that can be obtained from the detonation of an explosive mixture in a fully-filled detonation tube. By comparing the specific impulse-based efficiency values to the ideal efficiency values, we find that a fully-filled tube utilizes approximately 46% - 64% of the ideal energy computed from steps 1-2-3 of the Jacobs cycle (Table 3). This implies that approximately 36% - 54% of the explosive mixture's stored chemical energy that is available for doing mechanical work is not converted into impulse in a fully-filled detonation tube. It may be possible through improvements in the nozzle design to enhance the performance and recover some of this energy as additional impulse.

Mixture	$\epsilon_{Isp}/\epsilon_{Ideal}$
H_2-O_2	0.492
H_2-O_2-20%N_2	0.535
H_2-O_2-40%N_2	0.577
H_2-Air	0.632
C_2H_2-O_2	0.458
C_2H_2-O_2-20%N_2	0.488
C_2H_2-O_2-40%N_2	0.519
C_2H_2-O_2-60%N_2	0.560
C_2H_2-Air	0.607
C_2H_4-O_2	0.468
C_2H_4-O_2-20%N_2	0.495
C_2H_4-O_2-40%N_2	0.526
C_2H_4-O_2-60%N_2	0.569
C_2H_4-Air	0.623
C_3H_8-O_2	0.472
C_3H_8-O_2-20%N_2	0.497
C_3H_8-O_2-40%N_2	0.527
C_3H_8-O_2-60%N_2	0.569
C_3H_8-Air	0.638
$C_{10}H_{16}$-O_2	0.465
$C_{10}H_{16}$-O_2-20%N_2	0.489
$C_{10}H_{16}$-O_2-40%N_2	0.517
$C_{10}H_{16}$-O_2-60%N_2	0.558
$C_{10}H_{16}$-Air	0.634

Table 3 Ratio of specific impulse based efficiency to ideal efficiency values for several explosive mixtures at initial conditions of 100 kPa and 300 K.

Effect of Nitrogen dilution on detonation efficiency

Tables 1, 2, and 3 show an increase in efficiency as the diluent amount increases (Fig. 8). This trend can be related to the Chapman-Jouguet temperature which decreases with increasing nitrogen dilution. Fig. 9 shows the linear dependance of the efficiency ratio on the Chapman-Jouguet temperature, normalized by the initial temperature enabling predictions of detonation tube efficiency based on the mixture's detonation properties.

It should be noted that the efficiencies are higher for highly diluted mixtures yet the maximum possible work is lower. This may be observed by comparing the area under the processes connecting states 1, 2 and 3 of the Jacobs cycle for ethylene-oxygen-nitrogen mixtures at three different diluent amounts (Fig. 10).

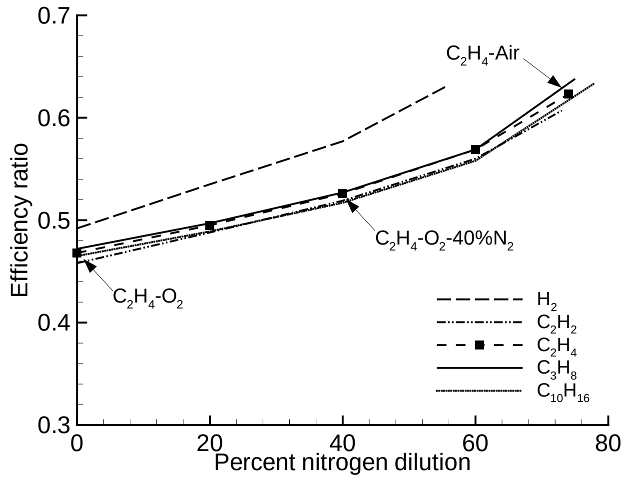

Fig. 8 Ratio of specific impulse-based efficiency to ideal efficiency values from Table 3 as a function of percent nitrogen dilution.

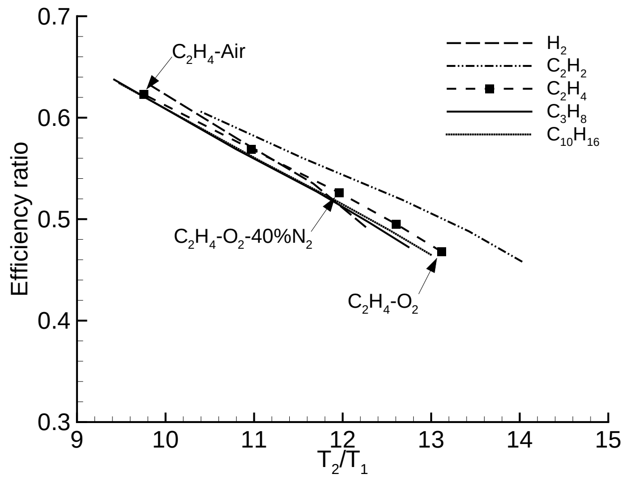

Fig. 9 Ratio of specific impulse-based efficiency to ideal efficiency values from Table 3 as a function of the normalized Chapman-Jouguet temperature.

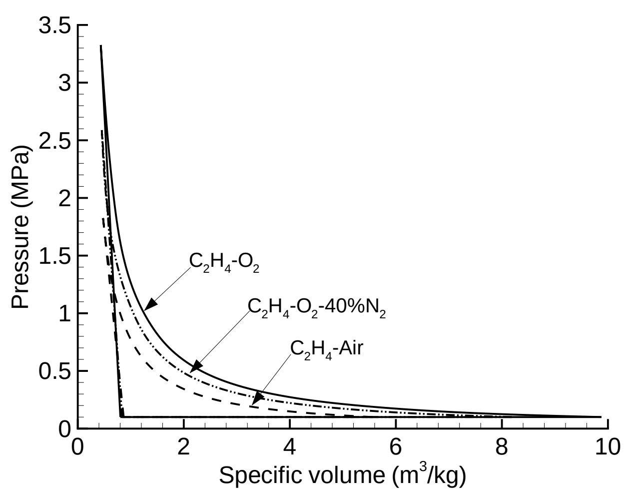

Fig. 10 Jacobs cycles for ethylene-oxygen-nitrogen mixtures.

Comparisons with partial-fill model

Our partial-fill model, Eq. (4) for $0.073 < V/V^{\circ} < 1$ and Eq. (5) for $0 < V/V^{\circ} < 0.073$, is compared to multi-cycle experiments by Schauer et al.[14] in hydrogen-air mixtures (Fig. 11). Data were obtained for a variety of tube dimensions, fill fractions, and cycle frequencies. Impulse and thrust measurements were taken with a damped thrust stand and we assume that multi-cycle operation is equivalent to a series of ideal single cycles.

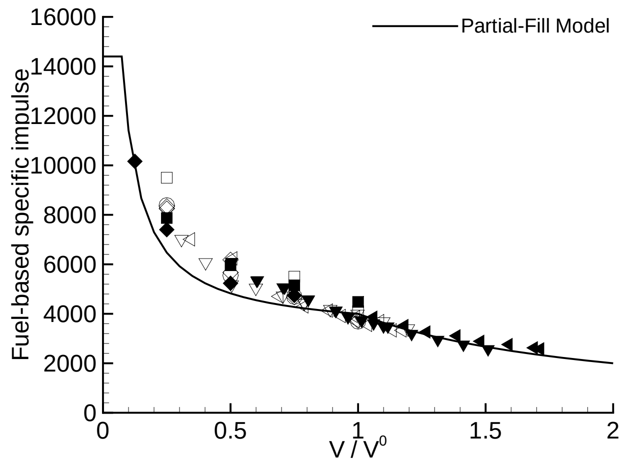

Fig. 11 Comparison of partial-fill model and multi-cycle experimental data.[14]

The fill fractions in Fig. 11 greater than 1 correspond to over-filling the detonation tube, and in this case, the impulse is reduced since only the mixture within the tube contributes to the impulse. This can be simply accounted for by computing the impulse as

$$\frac{I_{sp}}{I_{sp}^{\circ}} = \frac{V^{\circ}}{V} \tag{15}$$

when $V/V^{\circ} > 1$.

Li and Kailasanath[6] proposed a correlation for specific impulse of partially-filled tubes based on an exponential curve fit with data to the results of their numerical simulations

$$\frac{I_{spf}}{I_{spf}^{\circ}} = a - \frac{(a-1)}{exp\left(\dfrac{L^{\circ}/L - 1}{8}\right)} \ . \tag{16}$$

The constant a has values[6] between 3.2 and 3.5.

Equation (16) and the Gurney model (Eq. 6) in terms of fill fractions are compared with our partial-fill model (Fig. 12). All models predict zero impulse at a fill fraction of zero as expected. However, the Gurney model deviates significantly at small fill fractions when the specific impulse fraction is compared (Fig. 13). Our partial-fill model and the curve fit from the numerical simulations both tend to a constant specific impulse value in the limit of zero explosive mixture.

As the fill fraction tends to zero (or the N/C ratio tends to infinity), the impulse should scale linearly with fill fraction. This requires a finite slope near the origin of Fig. 12 resulting in a finite specific impulse value in Fig. 13. In this limit, the impulse predicted by the Gurney model scales as $I \sim V^{1/2}$ resulting in a slope that tends to infinity as the fill fraction tends to zero. The failure of the Gurney model in this limit is because the tamping mass (the inert gas) can never have a spatially uniform acceleration when it is infinite in extent. Wave processes are always important in determining the acceleration and the amount of material being accelerated continuously increases as the waves propagate away from the explosive portion of the tube. Although the Gurney model behavior in the limit of small fill fractions is incorrect, it correctly predicts the overall trend of impulse with fill fraction.

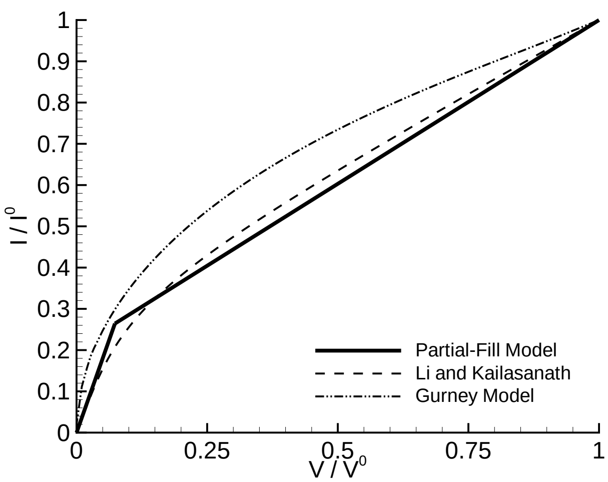

Fig. 12 Comparison of our partial-fill model to numerical simulations[6] and the Gurney model in terms of the impulse fraction. A value of 3.3 was used in evaluating Eq. (16).

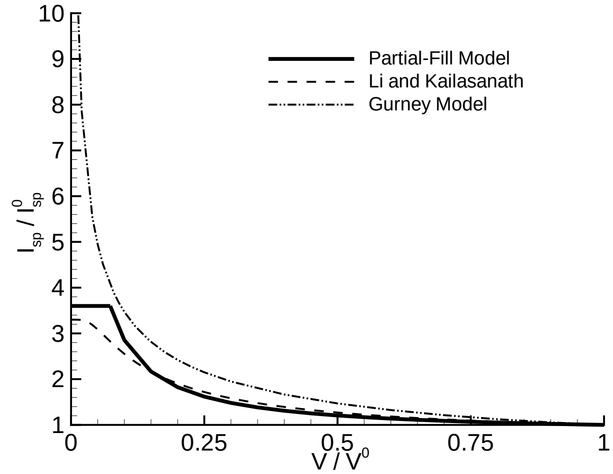

Fig. 13 Comparison of our partial-fill model to numerical simulations[6] and the Gurney model in terms of the specific impulse fraction. A value of 3.3 was used in evaluating Eq. (16).

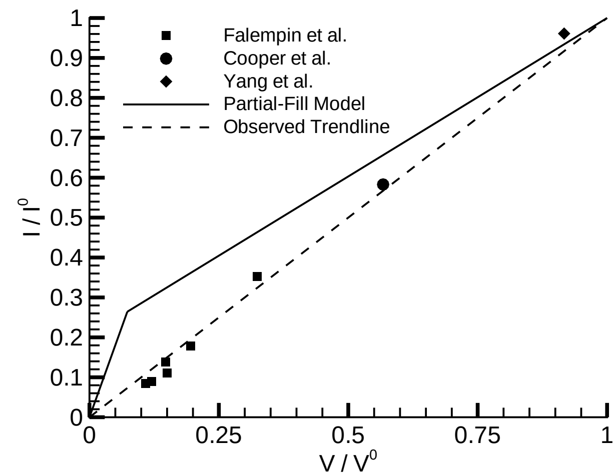

Fig. 14 Partial-fill model curve with data for detonation tubes of varying cross-sectional area.

Discussion of variable area tubes

The partial-fill model is extended to tube extensions of varying cross-sectional area such as diverging nozzles. In these cases, the percent fill is calculated by the ratio of the tube volume filled with the explosive mixture to the total tube volume. Experimental data[3,4] and numerical data[5] for straight diverging nozzles and other shapes in which the internal volume can be accurately calculated are plotted with the partial-fill model in terms of the impulse fraction (Fig. 14).

The disagreement between the experimental data and the model for cases of variable cross-sectional area tubes versus the cases of the constant-area cylindrical tubes (Fig. 5) implies the effect on impulse is not solely due to the tamping provided by the inert gas. A possible cause of the deviation may be attributed to the continuous area increase of a diverging nozzle. A series of reflected waves propagate back to the tube's thrust surface increasing the pressure relaxation rate at the thrust wall as compared with a straight extension that generates reflected waves (case B) only at the mixture interface and tube end. In fact, the data of Fig. 5 appear to follow a one-to-one relationship between fill fraction and impulse.

Conclusions

A simple model has been developed to predict the impulse in partially-filled detonation tubes. The model was based on interpretation of published experimental[1–4] and numerical[5,6] data. A piecewise linear correlation is found to adequately describe the existing data. The impulse increases with increasing fill fraction and the maximum value is obtained in a full tube. The specific impulse increases with decreasing fill fraction and the maximum value is obtained in the limit of vanishing explosive mixture amount. Another way to look at this is that the maximum specific impulse is obtained with an extension of inert gas that is very long compared to the extent of the explosive region. The Gurney model was utilized to correct the experimental data for the impulse increment that is a result of a finite diaphragm mass.

The maximum mechanical work obtainable from a detonation assuming ideal processes was calculated from the Jacobs cycle and found to be 27%–48% of the mixture's heat of combustion. However, only a portion of this work can be converted into impulse with a detonation tube. Using the Gurney model for an untamped explosive and the predicted[9] impulse, an effective Gurney energy has been determined for fully-filled detonation tubes. The impulse-based Gurney energy is between 12%–31% of the mixture's heat of combustion corresponding to 46%–64% of the ideal energy available to do mechanical work. Additionally, explosive mixtures with a higher amount of nitrogen dilution were found to have a higher efficiency. The efficiency was found to scale linearly with the Chapman-Jouguet temperature. Effective nozzle design may enable recovery of a greater fraction of the maximum possible energy through a more complete expansion of the detonation products.

Comparisons of the partial-fill model with single-cycle experimental and numerical data demonstrate that our simple model is effective in predicting the impulse for partially-filled detonation tubes of constant cross section. The model was compared to the measured average impulse of a multi-cycle PDE and reasonable agreement was obtained over a wide range of fill fractions. Comparisons with data obtained from tubes of varying cross-sectional areas indicate that diverging nozzles are less effective at increasing impulse than straight extensions of the same volume.

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