Mach Reflection of Detonation Waves

J. Meltzer,* J.E. Shepherd,† R. Akbar,* and A. Sabet,*
Rensselaer Polytechnic Institute, Troy , New York

Abstract

The diffraction of a nominally planar gaseous detonation at a wedge was investigated to determine the critical wedge angle for transition from regular to Mach reflection. Experiments were conducted in a square 83-mm cross-section detonation tube using stoichiometric mixtures of hydrogen-oxygen at 0.2 bars. Experimental results for the triple-point trajectory angle produced during Mach reflection were obtained using the smoke foil technique and are compared with analytic calculations made using three-shock theory and the oblique detonation polars. Measurements of the cell size behind the overdriven Mach stem are also reported. Both analytic and experimental results are compared with work from previous investigations to address apparent discrepancies in the existing literature.

---

* Graduate Student. Department of Aeronautical Engineering.

† Associate Professor. Department of Mechanical Engineering.

Progress in Astronautics and Aeronautics 153, 78-94, 1993.

Introduction

The motivation for this study is to resolve discrepancies in previous investigations regarding Mach reflection of a detonation wave. Whereas the diffraction of a shock wave by a wedge in a nonreactive gas has been the topic of considerable investigation, the reactive (detonation) case has received only limited attention in studies by Ong,[1] Gvozdeva and Predvoditeleva,[2] Manzhalei and Subbotin,[3] Gavrilenko et al.,[4] Gavrilenko and Prokhorov [5,6] and Edwards et al.[7] In each of these investigations, the detonation is given similar treatment as a nonreactive shock and standard[8] inviscid two- and three-shock theories applied to predict the triple-point trajectory and the critical angle for transition from regular to Mach reflection. Although detonations can be simplistically analyzed as shock waves followed by a thin chemical reaction zone, there are notable differences between the dynamics of nonreactive shocks and detonations. Foremost among these are the three-dimensional transverse instabilities which give rise to the cellular structure of the detonation front.[9] Furthermore, while the simplest model of a detonation, the Chapman-Jouguet (CJ) model, assumes that the reactions takes place instantly across a reactive shock, a detonation is known[10] to involve a coupling between an incident nonreactive shock followed by a chemical reaction zone. Additionally, expansion in the products leads to a Taylor wave, an expansion wave which follows the incident detonation. In all the analyses the transverse waves, reaction zone structure, and the Taylor wave are neglected. To reduce the complexity of the problem, this investigation, like those previous, also neglects these influences in the theoretical treatment.

In this investigation, experiments over a range of wedge angles were performed in a square cross-section detonation tube using stoichiometric hydrogen-oxygen mixtures at 0.2 bars. The results are compared to computations based on the three-shock model using realistic thermochemistry to predict the trajectory and transition angles. Both the experimental results, obtained using the smoke foil technique, and the calculated predictions are compared to the results of the previous studies to provide clarification.

Although a detonation wave is known[9] to posses a complex three-dimensional structure, in a one-dimensional sense, the Zel'dovich, von Neumann, Döring (ZND) model of a detonation idealizes the wave as a strong shock followed by a chemical reaction zone.[10] As such, the diffraction of a detonation would be expected to exhibit similar modes of behavior as a nonreactive shock as long as the reaction zone is thin compared to any other length scale present in the problem. For the case of Mach reflection, this implies that this simple treatment is valid when the Mach stem is much larger than the reaction zone thickness. The diffraction of nonreactive shocks has been investigated in depth in studies by von Neumann,[11] Law and Glass,[12] Ben-Dor and Glass,[13] Henderson,[14] and Hornung and Taylor.[15]

When a planar shock wave encounters a wedge, the incident shock is reflected by the wedge surface and the induced flow behind it is deflected by the wedge corner. Experimental investigations using interferometry and Schlieren techniques have shown that at least four types of reflections are possible depending on the wedge angle, Mach number of the incident wave, and initial conditions of the test gas. The reflections may be categorized into two principal modes, regular and Mach reflection. The latter of these cases may be further subdivided into single, complex, and double Mach reflections. All three types of Mach reflections share the similar features of a triple point with an incident shock and Mach stem, with the distinction arising from the the shape of the reflected wave. In our experimental investigation performed for the detonation case, no optical methods were available to determine the distinction in the type of Mach reflection. Therefore, the discussion of transition between modes of reflection presented here will be limited to the case of transition from regular to Mach reflection and the specific type of Mach reflection is not known.

A second noteworthy feature of the diffraction of a detonation by a wedge is that it also affords the opportunity to investigate the stability of overdriven detonations. The Mach stem produced during Mach reflection is an overdriven detonation, that is, a detonation which propagates with a velocity higher than the Chapman-Jouguet velocity. The Chapman-Jouguet velocity (U_{CJ}) is the calculated detonation wave speed that results in sonic flow in the products with respect to the detonation front. Freely propagating detonations usually travel at wave velocities (U) which are 95-98% of U_{CJ}. As the degree of overdrive ($U/U_{CJ} = M/M_{CJ}$) increases, the transverse wave spacing (cell size λ) decreases until the stability limit, at which point the transverse instabilities are damped out. In this investigation the stability limit is determined from smoke foil measurements of the cell size following the Mach stem.

The problem of the diffraction of a nominally planar detonation was first investigated by Ong[1] in his doctoral thesis at Michigan in 1955. Ong's investigation had two objectives: the first to predict the shape of the reflected wave generated during Mach reflection, the second to predict the critical angle for transition from regular to Mach reflection. Calculations of the shape of the reflected wave were made by applying perturbation techniques to the equations of motion in the psuedosteady frame. The critical wedge angle was determined by applying two-shock theory with constant specific heats (perfect gas) assumption to the case of regular reflection. Three-shock theory was applied to the case of Mach reflection to determine the trajectory of the triple point. To check his predictions, Ong performed experimental wedge studies in a rectangular detonation tube using an equimolar hydrogen-oxygen mixture at an initial pressure of 20 psia. Initiation of the detonation was through the deflagration to detonation transition (DDT) mechanism. Results, obtained experimentally from Schlieren photographs, showed the diffraction to be a single Mach reflection with the reflected wave being nearly circular. Whereas comparison is made with the reflected wave and trajectory angle, the experimental transition angle is not reported. Calculations performed by Ong may be in question due to a possible error in the values used for the initial conditions. In DDT, the pressure of the mixture ahead of the flame is increased by compression. The detonation is initiated at a pressure higher than the initial pressure, resulting in a change in the actual CJ conditions from those based on the original properties of the mixture.

The first published investigations were conducted by Russian researchers in the mid to late 1960s. Gvozdeva and Predvoditeleva[2] used Schlieren photography to investigate diffraction in methane-oxygen mixtures at 1 atm. Double- and triple-shock theories were used to determine the transition and trajectory angles. Calculations included realistic chemistry and were performed for both the frozen and equilibrium cases. Gvozdeva reports that the calculations did not compare well with the experiments, predicting regular reflection for some cases in which Mach reflection was observed experimentally. Also, Schlieren photographs for some cases indicate the reflected wave to be kinked, suggesting a complex or double Mach reflection process.

In the late 1970s Gavrilenko et. al.[4] studied the diffraction of a detonation wave as a method of producing overdriven detonations. Initial experiments were performed in a rectangular detonation tube with wedge inserts used to narrow the cross section. Results obtained from streak photographs were reported for the degree of overdrive as a function of wedge angle. Calculations were also performed using three-shock theory to predict the critical wedge angle. Gavrilenko reports that these calculations showed little dependence on mixture composition and initial pressure. A calculated critical wedge angle of 34 ± 0.4

deg. is reported for mixtures of $2H_2 + O_2$, $H_2 + O_2$, $4H_2 + O_2$, $C_2H_2 + 2.5O_2$, C_2H_2 + air (stoichiometric), $CO + 2O_2$, $CH_4 + 2O_2$, and CH_4 + air (stoichiometric) at pressures from 0.1 to 1 atm. Dilution with argon was predicted to increase the angle by approximately 3 deg.

Gavrilenko and Prokhorov[5,6] conducted additional experiments to determine the triple-point trajectory and the critical angle using single wedges in a rectangular detonation tube with results obtained by smoke foils, and Schlieren and streak photographs. Stoichiometric mixtures of hydrogen-oxygen and acetylene-oxygen at initial pressures ranging from 0.05 to 1 atm were tested and the critical angle for both mixtures found to be 40 ± 1 deg. Gavrilenko and Prokhorov do not report the exact pressures or cell sizes for which this was determined and make no mention of the sensitivity of the angle to either of these parameters. The maximum degree of overdrive is reported to be 1.3 and occurs at 38 deg, close to the critical angle. Gavrilenko states that the triple point motion is not self-similar, and that a unique correlation exists between the dimensions of the Mach stem and the detonation cell size of the incident wave. However, little detail is provided about this conclusion.

The most recent work, published by Edwards et al.[7] in 1984, had the objective of determining the effect of cell size on the diffraction process. Experiments were performed in a rectangular detonation tube with a wedge insert using stoichiometric mixtures of hydrogen-oxygen and acetylene-oxygen with different degrees of argon dilution. A range of incident cell sizes, 3-8 mm, were investigated by varying the mixture composition and initial pressure. Schlieren photographs and smoke foils were used to determine the trajectory of the triple point, critical wedge angle, and change in cell size as a function of overdrive. Barthel's[16] acoustic model for predicting the transverse wave spacing is used to estimate cell sizes behind the overdriven Mach stem and comparison is made with values measured from smoke foils. Barthel's model compares well up to an overdrive of 1.2, however, beyond this the model overpredicts the wave spacing. While the triple-point trajectory is shown to decrease with decreasing cell size, it is unclear whether this effect is due to the three-dimensional transverse wave structure or the change in initial pressure and Mach number of the incident wave.

In a later publication Nettleton[17] compares the experimental data from Edwards et al.[7] with predictions from Whitham's method for the diffraction of a shock at a compression corner and standard two- and three-shock theories. Whitham's method is an area-Mach number relationship developed for nonreactive shocks. It does not account for any interactions of pressure disturbances, generated behind the front with the diffracting wave.[18] Application of this method to a detonation using the equilibrium CJ model, is being studied at Rensselaer and will be presented in a later paper. As applied to the case of a compression corner, Whitham's method requires the existence of a Mach stem and is known[18] to give inexact predictions for the critical angle. Nettleton reports the two-shock calculation for $2H_2 + O_2 + Ar$ mixture yields a value of 65 deg, compared to the experimental value of 48 ± 2 deg. This is in disagreement with Gavrilenko and Prokhorov[5,6] who report a calculated value of 34 ± 0.4 deg and a measured value of 40 ± 1 deg.

Theory

The standard inviscid analysis of shock reflection considers the process as psuedosteady when viewed in a reference frame fixed with respect to the reflection or triple point. Figure 1 shows the cases of regular and Mach reflection as viewed in the laboratory and psuedosteady frames. The process is termed psuedosteady since the reflecting surface is not at rest with respect to the reflection or triple point, P in Fig. 1. In the case of regular reflection, the wall moves with the velocity of the incident flow U_1, while in Mach reflection, the wall not only translates but also appears to recede from the triple point with velocity U_w. In this frame, the standard[8] steady oblique shock relations in the form of a shock polar may be used to analyze the problem.

In regular reflection viewed in the psuedosteady frame, Fig. 1c, the initial flow U_1 is parallel to the wedge making an angle α with the incident shock I. After the shock, the flow is deflected by an amount θ_1. For an inviscid flow, the wedge establishes a boundary condition that the flow must be parallel to the surface. From this it is evident that the reflected wave R must be of sufficient strength to return the flow parallel to the wall, that is, $\theta_1 + \theta_2 = 0$. This situation is clearly illustrated by constructing the shock polar in the pressure-deflection (P-θ) plane. As shown in Fig. 2a, the flow returns to zero deflection when the reflected polar crosses the pressure axis.

From both analytic and experimental treatments of nonreactive shocks, it is known that for fixed initial conditions, as the wedge angle θ_w is decreased, the mode of reflection undergoes a transition from regular to Mach reflection. Mach reflection, as shown in Figs. 1b and 1d, is characterized by the appearance of a third shock, the Mach stem M, which is required to achieve the needed flow deflection. The position of the triple point grows linearly with time in a self-similar manner, along a straight trajectory at an angle χ to the wedge surface. A contact surface CS separates the gas processed by the incident and reflected shocks, regions 2 and 3, from that processed by the Mach stem, region 4. In the psuedosteady frame, Fig. 1d, the incoming flow is parallel to the triple-point trajectory at an angle α with the incident wave. As in regular reflection, it is deflected by an amount θ_1 by the incident wave and θ_2 by the reflected wave. However, in the case of Mach reflection, the deflection following the reflected wave is insufficient to return the flow to the original flow direction so that a third shock, the Mach stem, is needed to produce a deflection θ_3. The three-shock configuration is determined by matching the flow deflection angle and the pressure at the slipstream. From the matching condition across the slipstream $\theta_1 + \theta_2 = \theta_3$, that is the total deflection across the incident and reflected waves must be the same as the deflection across the Mach stem, and $P_3 = P_4$. The graphical solution to this matching process is given by finding the intersection of the polars for the incident and reflected shocks as shown in Fig. 2b.

Once the resulting wave configuration has been determined, the triple-point trajectory angle χ may be computed from geometric relations assuming that the Mach stem is straight and normal to the wall. Law and Glass[12] used this assumption in a three-shock analysis for nonreactive shocks in air and compared calculations for trajectory angle with experiments at a fixed Mach number. They concluded that the three-shock model predicted $\chi(\theta_w)$ accurately away from the regular-Mach reflection transition region, however, within 5 deg of the transition point, the experimental values of χ begin to substantially deviate from the three-shock predictions.

Using three-shock theory and the oblique detonation polars, theoretical predictions for the trajectory and critical angles were computed. A summary of the method developed by Sabet is presented here with a more detailed description given in Refs. 19 and 20. STANJAN,[21] a chemical equilibrium computer code, was run to determine the CJ conditions for the incident detonation. Using overdriven velocities, a modified version of STANJAN is used along with the Rankine-Hugoniot relations to construct the equilibrium detonation adiabat. A polynomial fit is obtained to describe the relationship $w_2=f(w_1)$, where w is the normal component of velocity and the subscripts denote the respective upstream and downstream states. This relation is used to numerically solve the oblique shock relations at a specified angle of incidence α. From this, the overdriven detonation polar is constructed in the P-θ plane. The reflected polar is generated in a similar fashion by computing equilibrium shock adiabats for gas initially at CJ conditions behind the incident (reactive) wave. The oblique detonation polar for $\alpha = 60$ deg is shown in Fig. 3. The polars start at the same point, the CJ point corresponding to state 2 in Fig. 1d. The intersection point corresponds to the matching condition across the slipstream.

Knowing the pressure P_4, in the region behind the Mach stem, the detonation adiabat may be used to find an estimate of the normal velocities across the Mach stem, w_{1m} and w_4. The trajectory angle χ is then computed solving the quadratic equation

$$(\tan \chi)^2 - \frac{\left(1 - \frac{w_4}{w_{1m}}\right)}{\tan \theta_3} \tan\chi + \frac{w_4}{w_{1m}} = 0 \qquad (1)$$

which can be obtained from geometric considerations. The value of w_{1m} may be improved using the computed χ,

$$w_{1m} = U_1 \cos\chi \qquad (2)$$

and an iterative process is used to determine an acceptable χ. The final step is to determine the wedge angle corresponding to the specified α and calculated χ,

$$\theta_w = \frac{\pi}{2} - \alpha - \chi \qquad (3)$$

To check the validity of our method, some nonreactive shock cases were computed and the results compared with previous experimental (Henderson[14]) and theoretical results (Ben-Dor and Glass[13]). As shown in Fig. 4, our results compared extremely well with both sets of results.

Experimental

Experiments[19] were performed in a detonation tube 4.88 m in length using wooden wedges varying from 10 to 50 deg in 5-deg increments. The tube had a 83-mm-square cross-section with 6.35-mm radius rounded corners. The wedges, which were mounted to the end flange of the tube, were fitted with a 3.2 mm-thick aluminum face milled to a sharp leading edge so as to rest flat on the bottom of the tube. Smoke foils were placed along the side of the wedge to record the triple-point trajectory and the cell structure behind the incident wave and Mach stem. Foils were cut from 0.635 mm-thick aluminum sheet and prepared over an open kerosene lantern. It was determined that the best traces, i.e., with the most detail, were obtained when the foils were lightly sooted for approximately 10 min.

Stoichiometric mixtures of hydrogen-oxygen at an initial pressure of approximately 0.2 bars were used throughout the experiments. Initiation of the detonation was achieved through use of an exploding wire system. Confirmation was obtained by comparing the measured wave velocity and pressure with known CJ conditions. Two PCB model 113A21 piezoelectric pressure transducers were used to obtain pressure traces and to time the speed of the incident detonation. For the test mixture, the calculated[21] CJ pressure was 3.63 bar and the velocity was 2750 m/s, yielding a Mach number equal to 5.1. The measured wave velocity was consistently 95-98% of the calculated CJ velocity.

Results

Results for the triple-point trajectory angle and cell sizes were determined from smoke foils. The regularity of the cell structure behind the incident wave was moderate, with the average cell size ranging from 7-8 mm, which agrees well with published data.[22] Interpretation of the foils was hindered by a loss in quality due to the irregular interference of the slapping waves. Identification of the triple-point trajectory was made by noting the change in cell size between the incident wave and Mach stem on the side foil. It was also observed that cells behind the Mach stem were aligned parallel to the wedge, while those behind the incident wave were aligned along the tube axis. Measurements of the cell size behind the overdriven Mach stem were taken from foils placed on the wedge surface. Direct measurements were made by viewing the foils under a microscope.

An example of a smoke foil obtained for a 30-deg wedge is shown in Fig. 5, while the results of the triple-point trajectory and the overdriven cell size are summarized in Table 1. For the 10 and 15-deg wedges, the degree of overdrive is slight and the average ratio of the overdriven cell size to the incident cell size, λ_M/λ_{CJ}, ranged from 0.58 to 0.82. The limited reduction in cell size led to difficulty in measuring the trajectory.

The clearest results for the triple point trajectory were obtained for the 20-, 25-, and 30-deg cases. For these cases, the average ratio of the cell sizes, λ_M/λ_{CJ}, ranged from 0.26 to 0.48 with the cells behind the Mach stem being large enough to see clearly by eye, yet having sufficient reduction to be easily distinguishable from the incident cells. The higher overdrive at these angles greatly increases the regularity of the overdriven cell structure as seen on the front foils.

At 35 deg the trajectory angle is small, 3 ± 1 deg, and the cell structure behind the Mach stem on the side foil is no longer visible by the naked eye. At 40 deg the Mach stem has become marginally stable. The front foil reveals only scattered patches of cells that appear, die out, and reappear. Viewing the region under the trajectory on the side foil reveals the same evidence, intermittent patches of cells with the structure dying out. Where cells existed, λ_M/λ_{CJ} was 0.09. At 45 and 50 deg there is no evidence of a triple-point trajectory or of overdriven cells on the front foil. From this, it was concluded that the critical angle for transition from Mach to regular reflection is between 40 and 45 deg.

Figure 6 shows a plot of the trajectory angle χ as a function of wedge angle. The large error bars for the 10- and 15-deg cases reflect the difficulty in determining the correct angle for these cases. For the remaining cases the trajectory angle was distinguishable to ± 2 deg. The zero angle data point at 45 deg indicates regular reflection.

The ratio of the overdriven to incident cell size as a function of wedge angle is plotted in Fig. 7. The data points represent the average overdriven cell sizes normalized by the average incident cell size, 7.5 mm. The range of the error bars correspond to the range of overdriven cell sizes measured on the front foil. The small error bars for the 30-, 35-, and 40-deg, wedges indicate the high regularity of the overdriven cell structure. The 40-deg angle was taken as the stability limit. The stability limit does not represent a condition of zero cell size, rather the condition at which the overdrive is sufficient to cause damping of the natural transverse disturbances.

Discussion

The three-shock method described previously was used to compute triple-point trajectory angles for stoichiometric hydrogen-oxygen mixtures at 0, 30 and 70% dilution with argon. The results, shown in Fig. 8, demonstrate that the predicted trajectory is not very sensitive to the degree of dilution. The critical angle was determined as the extrapolated wedge angle at which $\chi = 0$. From the figure it was calculated to be approximately 34 deg, in excellent agreement with the value determined by Gavrilenko.[4] Nettleton[17] reports calculated critical angles of 50 deg for argon-diluted $2C_2H_2 + 5O_2$ mixtures and 65 deg for argon-diluted $2H_2 + O_2$ mixtures. These results were obtained with nonreactive shock dynamics.[18] A more appropriate reactive flow version of Whitham's method must be carried out before the validity of this technique can be judged. Such a comparison is being worked on by the present authors.

In Fig. 6 the experimentally determined values for χ are compared with computed values for the 0% dilution case. the experimental critical angle is between 40 and 45 deg for the case studied in the present investigation. Our three-shock analysis consistently underpredicts the triple point trajectory χ, and the critical angle, by 6-10 deg. Our experimental results are similar to those of Gvozdeva[2] and Gavrilenko and Prokhorov.[6] They determined a critical angle of about 35 deg for stochiometric CH_4-O_2 and 40 deg for stochiometric H_2-O_2 and C_2H_2-O_2 mixtures. It has been reported[23] that similar detonation diffraction phenomena depend strongly on cellular regularity. In addition, we also anticipate the cell width to be an important parameter. We would only expect a self-similar diffraction process if the cell width is much smaller than the Mach stem. A limited exploration of these issues was performed in the study by Edwards et al.[7], who tested three mixtures with various cell widths. The critical wedge angle $\theta_{w,crit}$ was found to be different for all three mixtures. It is interesting that the three mixtures correspond to three different levels of cellular structure irregularity. These are, in order of increasing regularity: $C_2H_2 + O_2$, $\theta_{w,crit} = 33$ deg; $2H_2 + O_2 + Ar$, $\theta_{w,crit} = 45$ deg and $2H_2 + O_2 + 70\%Ar$, $\theta_{w,crit} = 48$ deg. Further tests are needed to clarify this issue. In addition, a systematic study of the triple point trajectories as a function of cell width would also be useful.

The comparison shows that for a detonation the observed Mach reflection regime extends into the theoretical region of regular reflection. This effect, which is the same as reported by Gvozdeva,[2] is opposite of the effect observed in the diffraction of nonreactive shocks. In the nonreactive shock case, regular reflection has been observed to occur in the theoretical Mach reflection region. This has been attributed to viscous effects by Hornung and Taylor.[15] Hornung and Taylor's argument is as follows. For regular reflection in the psuedosteady frame, the velocity of the flow following the reflected wave will be less than the velocity of the wall, which moves at the same speed as the incident flow. Considering the flow as viscous, this leads to a boundary layer with a negative displacement thickness, resulting in an apparent flow into the wall. The deflection condition for the viscous case requires that the flow following the reflected wave must be turned parallel to the effective wall. As a result, the deflection of the flow after the reflected wave is less than in the inviscid case by an amount ε and the deflection condition becomes $\theta_1 + \theta_2 = \varepsilon$. The maximum incident flow deflection which can be returned by the reflected wave occurs at a lower wedge angle than in the inviscid case. That is, the critical wedge angle, θ_w, is lower in the viscous case than in the inviscid case. Hornung and Taylor[15] performed a careful experimental study of the viscous effect and concluded that the critical angle for Mach reflection could be decreased by as much as 7 deg. The flow behind the detonation should be expected to experience the same type of viscous effects as in the nonreactive shock case. This would imply that there should be a lowering of the critical wedge angle, up to 7 deg from the calculated inviscid value, extending regular reflection into the Mach reflection regime. This is opposite of the experimental observations, in which the critical wedge angle is observed to be 6-10 deg higher than the predicted value.

Whereas no definite conclusion has been obtained, we propose that the discrepancy between the three-shock theory and the experimental data is due to the three-dimensional instability wave structure of the detonation. The three-shock model is a one-dimensional treatment and does not account for interaction of the transverse structure with the diffracting wave. The finite thickness of the reaction zone could be another reason for the failure of the three-shock analysis to correctly predict the triple-point trajectory and critical angle. In the present experiments, the reaction zone length behind the undisturbed detonation is calculated to be 0.24 mm and is about 0.05 mm behind the Mach stem at an overdrive of $U/U_{CJ} = 1.3$, near the critical angle. For wedge angles less than 30 deg, the Mach stem is greater than 10 mm high at the end of the wedge, suggesting that the reaction zone effects should be small for $\theta_w < 30$ deg. However, if the nominal cell size of 7.5 mm is used as the effective reaction zone length, then it is not possible to conclude that effects of the detonation structure are negligible. Experiments using a systematic variation of the incident wave cell size are needed to investigate this issue further. One experiment which might give further insight would be to test $2H_2 + O_2 + 7Ar$ mixtures at high initial pressures, which have very regular and small amplitude instability waves.[22]

Further investigation including Schlieren photography is also needed to determine whether the detonation experiences single, complex, or double Mach reflection. Although the idealized three-shock theory does not account for any type other than single Mach reflection, based on evidence from nonreactive shocks it is not felt that the possible presence of complex or double reflections can account for the discrepancy observed in the detonation case. Ben-Dor and Glass[13] compared experimental results for the trajectory angle as a function of wedge angle and Mach number for shocks in N_2 with predictions calculated using a three-shock theory and polar method developed by Law and Glass.[12] The comparison showed good agreement to ± 2 deg over the range $20 < \theta_w < 40$ deg and $1 < M < 8$. This is despite the fact that for a fixed Mach number, for example $M = 5$ as in our detonation case, the shock was observed experimentally to undergo single Mach reflection for $\theta_w < 14$ deg , complex Mach reflection for $14 < \theta_w < 25$ deg, and double Mach reflection for $25 < \theta_w < 48$ deg.

Another shortcoming of our analysis to predict χ may be the assumption that the Mach stem is straight and normal to the wall. As noted by Hornung,[24] experimental evidence for the Mach reflection of shocks has shown that the slipstream is curled toward the Mach stem. This is due to a wall pressure gradient which is formed from the difference in stagnation pressures for flows on either side of the slipstream striking the wedge. This wall pressure gradient can also result in a strong wall jet which may interact with the Mach stem leading to curvature.

Analytic predictions for the overdriven cell size ratio as a function of overdrive were made using a computer code[25] which uses the ZND model to resolve the reaction zone structure. A comparison with experimental data, shown in Fig. 7, indicates that at lower overdrives the results are close but that the calculated values slightly overpredict values at higher angles. Without direct measurements available, the overdriven velocities at each angle were determined from geometric relations using the experimental values for χ. The uncertainty in the χ data results in the displayed uncertainties in overdrive. The effect of the overdrive on the cell size has been reported previously by various authors.[26-29] In particular, Desbordes et al.[27-29] overdrive the detonation by an abrupt transition from a stochiometric mixture to a lean mixture. The highest overdrive reported by them[29] is D/D_{CJ} of about 1.3 (oxygen-acetylene mixture).

Conclusion

The critical angle for transition from regular to Mach reflection for a stoichiometric hydrogen-oxygen mixture at 0.2 bar was determined experimentally to be 40-45 deg. This is above the theoretical value of 34 deg computed using three-shock theory and the oblique detonation polars. The experimental results of this investigation are consistent with results reported by Gavrilenko et al.[4-6] and are bracketed by the values reported by Edwards et al.[7]

Our three-shock theoretical analysis agrees with that of Gavrilenko et al.[4-6] and disagrees with the approximate analysis of Nettleton.[17] Unlike experiments with nonreactive shocks in which the regular reflection is observed to extend below the theoretical critical wedge angle, in the detonation case the region of Mach reflection is observed to extend into the theoretical regular reflection regime. This is most probably due to inadequacies of the idealized three-shock theory when applied to the detonation case. In particular, the three-dimensional detonation transverse wave structure is expected to have a substantial influence on the diffraction process. Further experimental investigation, including Schlieren photography, is needed to gain a better understanding of the problem.

The diffraction of a detonation wave by a wedge provides an effective way of obtaining results for the cell size in an overdriven detonation. Measurements of overdriven cell sizes taken from the smoke foils are in line with values predicted by the ZND model and compared well with other experimental values reported in previous investigations. From our experiments the stability limit was found to occur at a wedge angle of about 40 deg, at a calculated overdrive of approximately $M/M_{CJ} = 1.36$.

Acknowledgments

Funding for this research was provided by a National Science Foundation Engineering Research Equipment Grant MSM - 8806189. Partial support was also provided by Lawrence Livermore National Laboratories under Contract B055778. We would like to thank J. H. S. Lee, R. Knystautas and D. Desbordes for their useful comments.

References

[1] Ong, R. S., "On the Interaction of a Chapman-Jouget Detonation Wave with a Wedge," PhD. Thesis, University of Michigan, 1955.

[2] Gvozdeva, L. G., and Predvoditeleva, O. A., "Triple Configurations of Detonation Waves in Gases," Fizika Goreniya i Vzryva, Vol. 5, No. 4, 1969, pp. 451-461.

[3] Manzhalei, V. J., and Subbotin, V. A., "Stability of an Overcompressed Detonation," Combustion, Explosion, and Shock Waves, Vol. 12, No. 6, 1977, pp. 819-825.

[4] Gavrilenko, T. P., Nikolaev, Y. A., and Topchiyan, M. R., "Supercompressed Detonation Waves," Combustion, Explosion, and Shock Waves, Vol. 15, No. 5, 1980, pp. 659-692.

[5] Gavrilenko, T. P., and Prokhorov E. S., "Compressed Detonation Wave in a Real Gas," Combustion, Explosion, and Shock Waves, Vol 17, No. 6, 1982, pp. 689-692.

[6] Gavrilenko, T. P., and Prokhorov E. S., "Overdriven Gaseous Detonations," Shock Waves, Explosions and Detonations: Progress in Astronautics and Aeronautics, edited by M. Summerfield, Vol. 87, AIAA, New York, 1983, pp. 244-250.

[7] Edwards, D. H., Walker, J. R., and Nettleton, M. A., "On the Propagation of Detonation Waves Along Wedges," Archivum Combustion, Vol. 4, No. 3, 1984, pp. 197-209.

[8] Thompson, P. A., Compressible Fluid Dynamics, McGraw-Hill, New York, 1972.

[9] Lee, J. H. S., "Dynamic Parameters of Gaseous Detonations," Annual Review of Fluid Mechanics, Vol. 16, 1984, pp. 311-336.

[10] Fickett, W., and Davis, W. C., Detonation, University of California Press, Berkley, CA, 1979.

[11]von Neumann, J., Collected Works. Pergamon, Oxford, England, 1963.

[12]Law, C. K., and Glass, I. I., "Diffraction of Strong Shock Waves by a Sharp Compressive Corner," C. A. S. I. Transactions, Vol. 4, 1971, pp. 2-12.

[13]Ben-Dor, G., and Glass, I. I., "Domains and Boundaries of Non Stationary Oblique Shock Wave Reflexions. Part 1. Diatomic Gas," Journal of Fluid Mechanics, Vol. 92, 1979, pp. 459-496.

[14]Henderson, L. F., "On the Whitham Theory of Shock Wave Diffraction at a Convex Corner," Journal of Fluid Mechanics, Vol . 99, 1980, pp. 801-811.

[15]Hornung, H., and Taylor, J. R., "Transition from Regular to Mach Reflection of Shock Waves. Part 1. The Effect of Viscosity in the Psuedosteady Case," Journal of Fluid Mechanics, Vol. 123, 1982, pp. 143-153.

[16]Barthel, H. O., "Predicted Spacings in Hydrogen-Oxygen-Argon Detonations," The Physics of Fluids, Vol. 17, No. 8, 1974, pp. 1547-1553.

[17]Nettleton, M. A., Gaseous Detonations: Their Nature and Control, Chapman and Hall, New York, 1987.

[18]Whitham, G. B., "A New Approach to the Problem of Shock Dynamics. Part 1. Two Dimensional Problems," Journal of Fluid Mechanics, Vol. 2, 1957 pp. 145-171.

[19]Meltzer, J. S., "The Diffraction of a Detonation by a Wedge," Master's Thesis, Rensselaer Polytechnic Institute, Troy, NY, Dec. 1990.

[20]Sabet, A. I., "Investigation of Equilibrium and Chemical Kinetic Behavior of Detonation Waves in Hydrogen-Air Mixtures and an Evaluation of the Oblique Detonation Wave as the Combustor for a Scramjet," Master's Thesis, Rensselaer Polytechnic Institute, Troy, NY, May 1990.

[21]Reynolds, W. C., "The Element Potential Method for Chemical Equilibrium Analysis: Implementation in the Interactive Program STANJAN; Version 3," Department of Mechanical Engineering, Stanford Univ., Palo Alto, CA, Jan. 1986.

[22]Strehlow, R. A., and Engel, C. D., "Transverse Waves in Detonations II. Structure and Spacing in H_2-O_2, C_2H_2-O_2, C_2H_4-O_2 and CH_4-O_2 Systems," AIAA Journal, Vol. 17, No. 3, 1969, pp. 492- 496.

[23]Shepherd, J. E., Moen, I. O., Murray, S.B., and Thibault, P. A., "Analyses of the Cellular Structure of Detonation," 21st Symposium [International] on Combustion, The Combustion Institute, Pittsburgh, PA, 1986, pp. 1649-1658.

[24]Hornung, H., "Regular and Mach Reflections of Shock Waves," Annual Review of Fluid Mechanics, Vol. 18, 1986, pp. 35-38.

[25]Shepherd, J. E., "Chemical Kinetics of Hydrogen-Air Diluent Detonations," Dynamics of Explosions: Progress in Astronautics and Aeronautics edited by Martin Summerfield, Vol. 106, AIAA, New York, 1986, pp. 263-293.

[26]Vasiliev, A. A., and Nikolaev, Y., "Theoretical Model of a Detonation Cell," Acta Astronautica, Vol. 5, Pergamon Press, New York, 1978, pp. 983-996.

[27]Desbordes, D., and Vachon, M., " Critical Diameter of Diffraction for Strong Plane Detonations," Dynamics of Explosions: Progress in Astronautics and Aeronautics, edited by M. Summerfield, Vol. 106, AIAA, New York, 1986, pp. 131-143.

[28]Desbordes, D., "Transmission of Overdriven Plane Detonations: Critical Diameter as a Function of Cell Regularity and Size," Dynamics of Explosions: Progress in Astronautics and Aeronautics, edited by M. Summerfield, Vol. 114, AIAA, New York, 1988, pp. 170-

[29]Desbordes, D., and Lannoy, A., "Effects of a Negative Step of Fuel Concentration on Critical Diameter of Diffraction of a Detonation," Dynamics of Detonations and Explosions: Detonations: Progress in Astronautics and Aeronautics, edited by A.R. Seebass, Vol. 133, AIAA, New York, 1990, pp. 170-186.

Fig. 1 Reflection of a shock wave incident on a wedge viewed in the lab frame: a) Regular reflection; b) Single Mach reflection; c) regular and d) Mach reflections viewed in the psuedosteady frame.

Fig. 2 Idealized shock polars illustrating a) regular and b) Mach reflections.

Fig. 3 Oblique detonation polar for $2H_2 + O_2$ mixture at 0.2 bar with α = 60 deg.

Fig. 4 Comparison of calculated trajectory angle for nonreactive shocks; Sabet method is compared with results from Henderson[14] and Ben-Dor and Glass.[13]

Fig. 5 Smoke foils for 30-deg wedge: a) side foil; b) front foil.

Fig. 6 Comparison of theoretical and experimental trajectory angles for detonations in $2H_2 + O_2$ mixtures at 0.2 bar.

Fig. 7 Comparison of measured overdriven cell size ratio with ZND calculations.

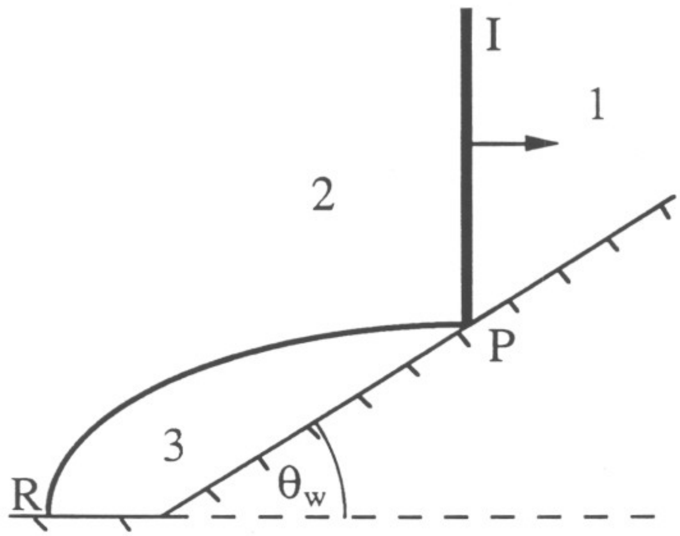
Fig. 8 Theoretical trajectory angles for CJ detonation in $2H_2 + O_2$ mixtures at 0.15 bar diluted with argon: □ 0% Ar; ○ 50% Ar; ◇ 70% Ar.

Table 1. Summary of experimental results for trajectory angle and overdriven cell size.

Wedge angle θ_w (deg)	Trajectory angle χ (deg)	Overdriven cell size λ_M (mm)	Cell size ratio, S λ_M/λ_{CJ}
10	17 ± 3	5.1 - 7.1	0.68 - 0.95
15	15 ± 3	3.0 - 5.6	0.41 - 0.75
20	12 ± 2	2.5 - 4.6	0.34 - 0.61
25	7 ± 1	2.7 - 3.2	0.36 - 0.42
30	6 ± 1	1.8 - 2.0	0.24 - 0.27
35	3 ± 1	1.3 - 1.5	0.17 - 0.20
40	2.5 ± 0.5	0.5 - 0.8	0.07 - 0.10
45	*	*	*

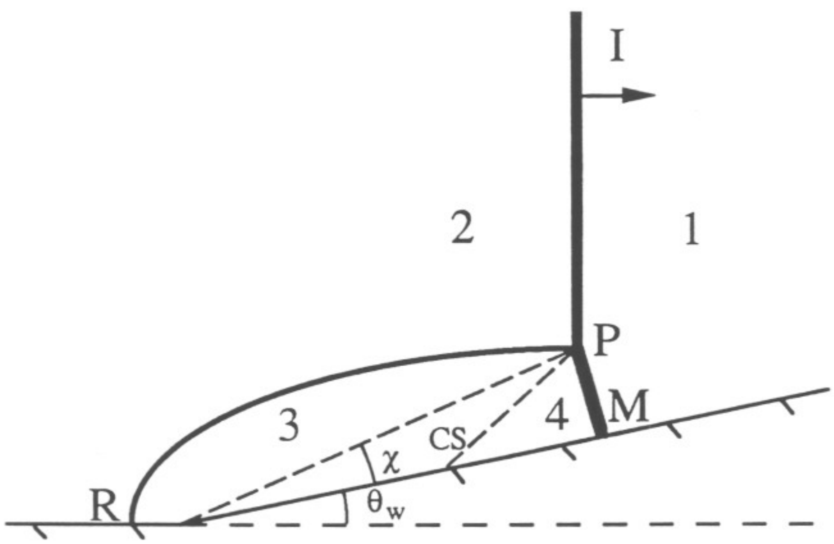
* indicates regular reflection

Meltzer *et al.* Figure 1a.

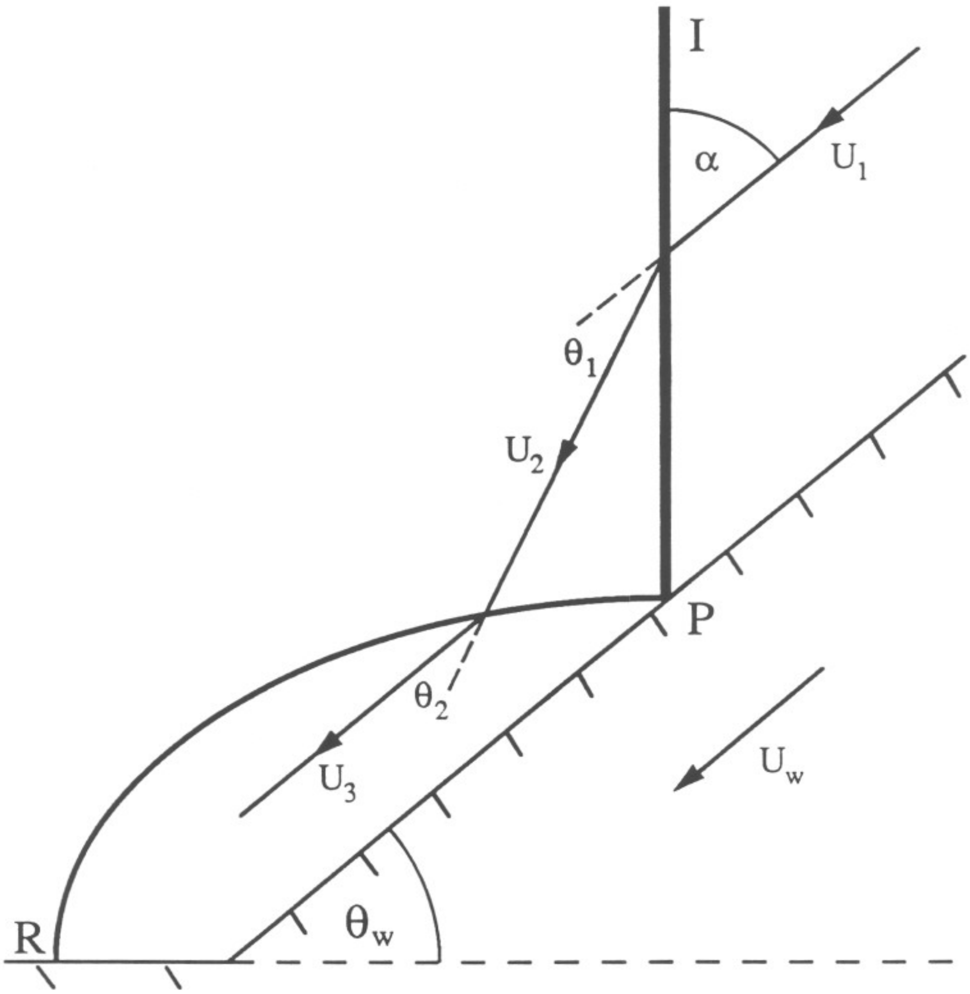


(a)

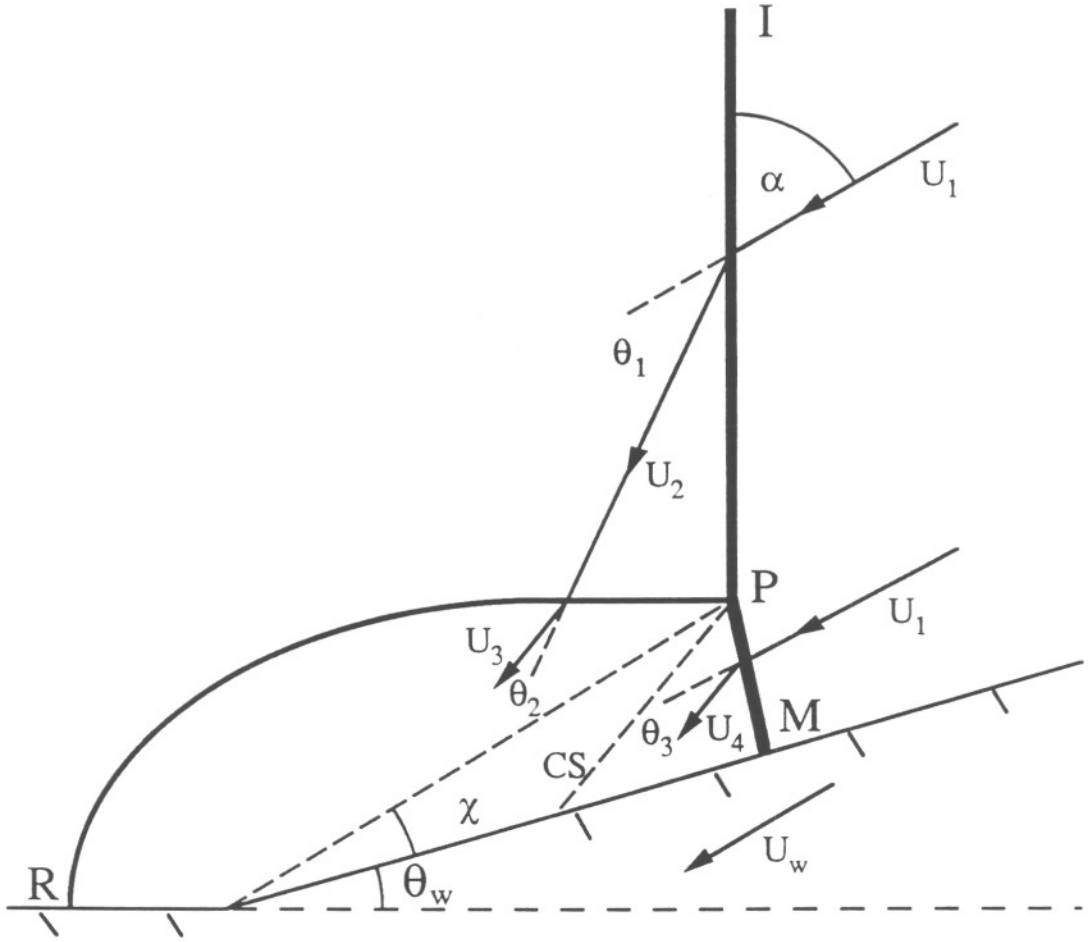
Meltzer *et al.* Figure 1b.



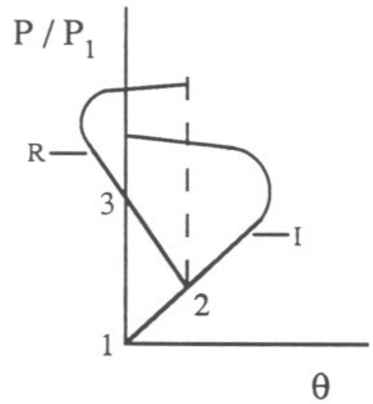
(b)



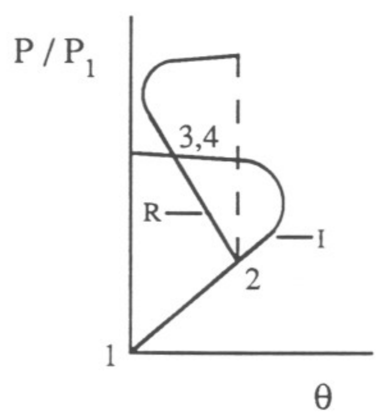
(c)



(d)

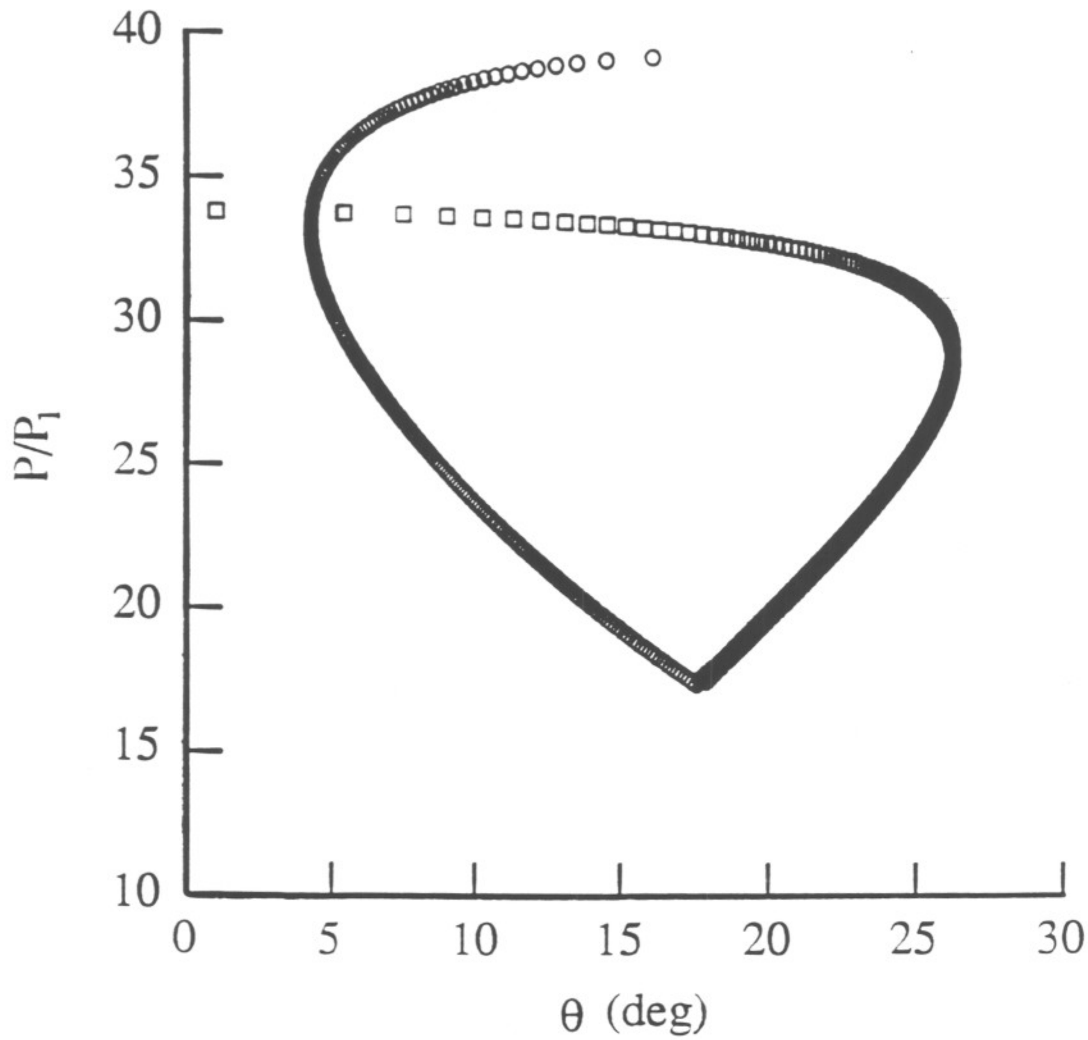


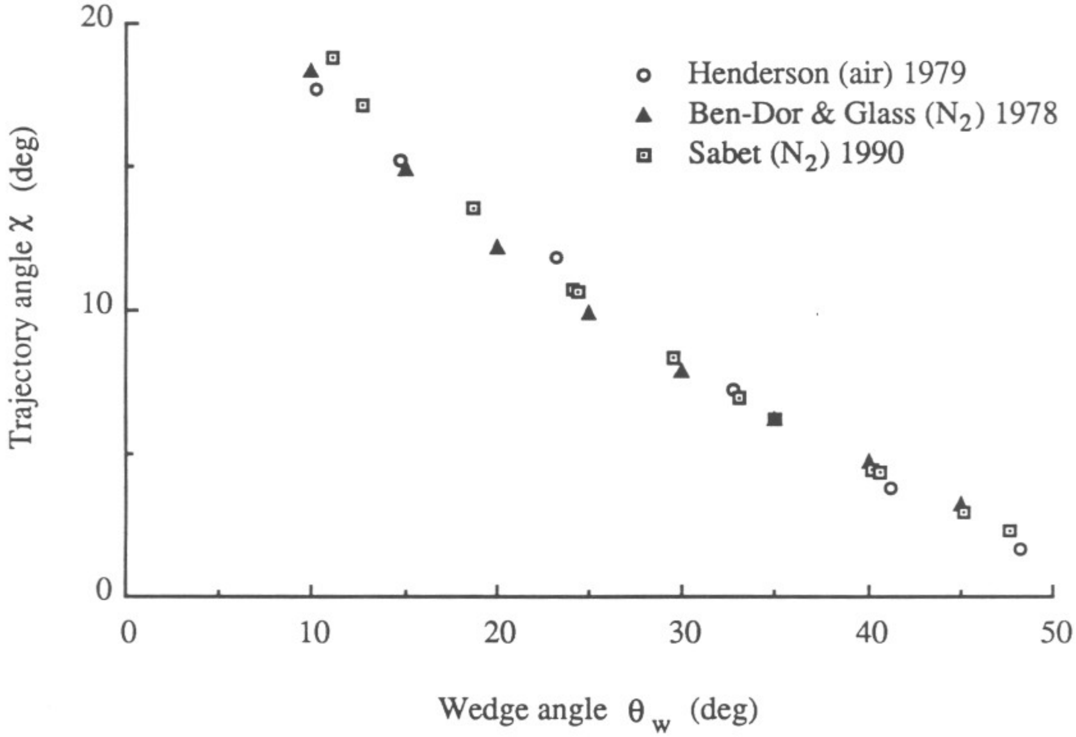
(a)



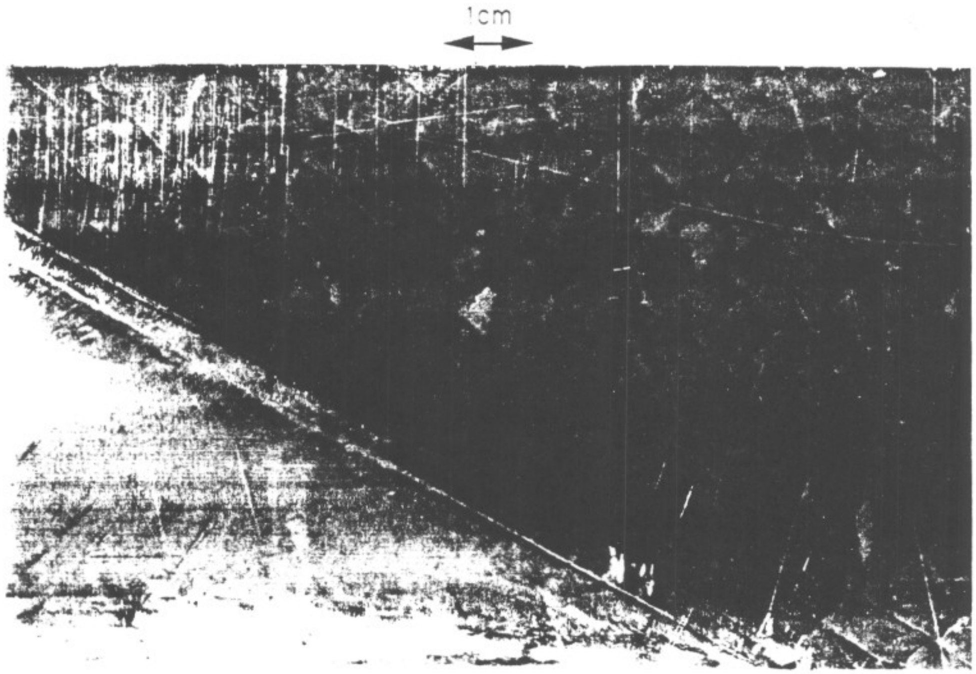
(b)

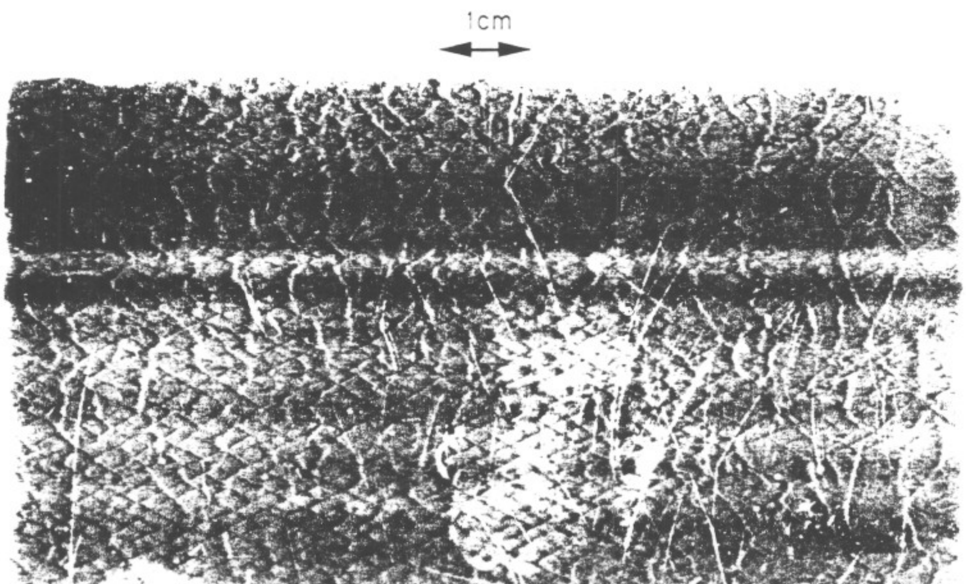
Meltzer et al. Figure 3



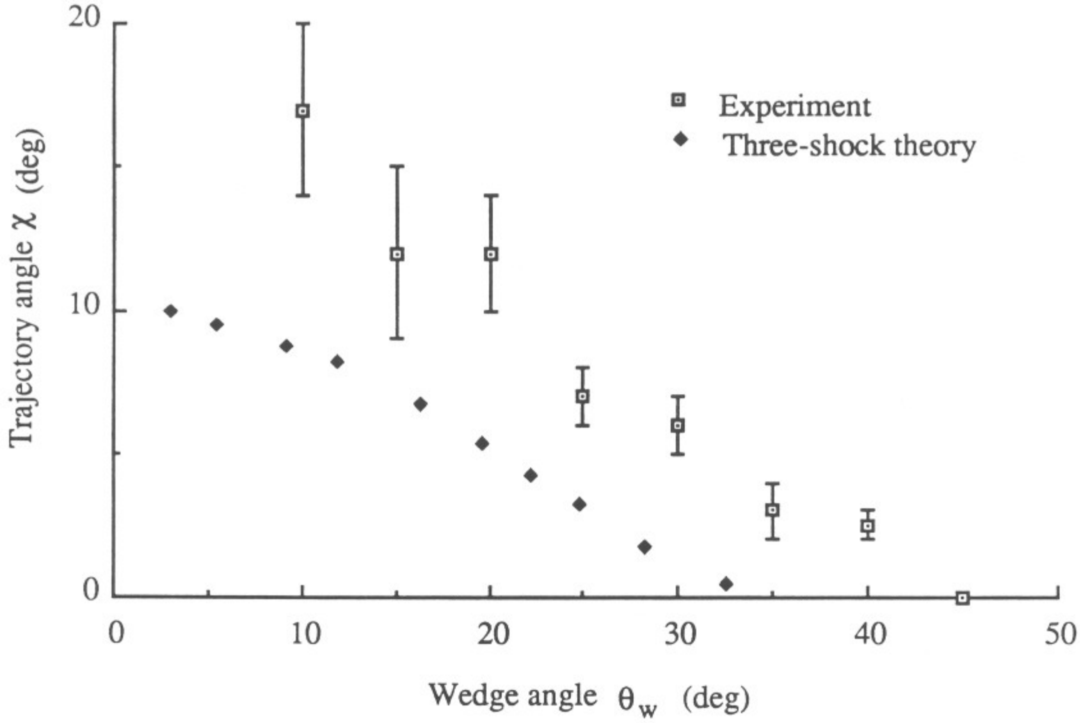


20
10
0
0
10
20
30
40
50
Trajectory angle χ (deg)
Wedge angle θ w (deg)
Henderson (air) 1979
Ben-Dor & Glass (N2) 1978
Sabet (N2) 1990

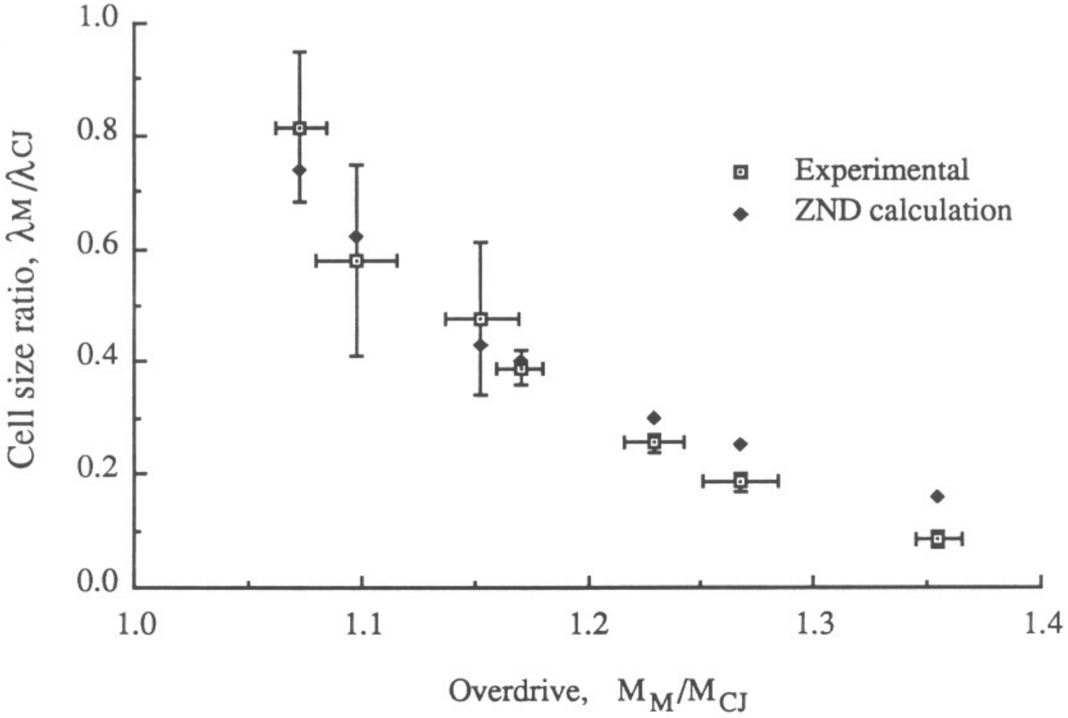

1cm


1cm

Meltzer et al. Figure 6



Meltzer et al. Figure 7



Meltzer et al. Figure 8