Goals of the experiment

In this experiment you will assemble and align a schlieren/shadowgraph setup from components in the Cann Laboratory and use it to visualize phenomena in fluids and solids using a high-speed camera. Upon completion of this experiment, you should:

- Have a fundamental understanding of the theory of schlieren and shadowgraph visualization techniques
- Have a working knowledge of schlieren and shadowgraph setups including placement and alignment of optics and practical issues involved with the individual components involved: mirrors, light source, cutoff, focusing optics, and camera. Given components, you should be able to set up your own schlieren or shadowgraph system
- Understand what types of phenomena are good candidates for schlieren and shadowgraph visualization

Basic principles

This project deals with schlieren and shadowgraph visualization techniques, which translate phase speed differences in light, invisible to the eye, into changes in intensity which we perceive as regions of light and dark. A brief discussion of these techniques is presented here; for a more complete description the reader is referred to the famous book by Settles [1]. Change in phase speed as light passes through a transparent medium is called refraction. The refractive index $n = c_0/c$ of a medium describes the change in phase speed where $c$ is the speed of light in the medium and $c_0$ is the speed of light in a vacuum. For gases the refractive index is linearly dependent on the gas density according to the Gladstone-Dale relation:

$$n = K \rho + 1$$

where $K$ is the Gladstone-Dale constant and $\rho$ is the gas density. $K$ is dependent on gas composition, temperature, and the wavelength of the light passing through the medium. In gases $K$ is typically between $0.1 \times 10^{-3}$ and $1.5 \times 10^{-3}$ (kg/m$^3$)$^{-1}$. $K\rho$ is therefore much smaller than 1, so the refractive index varies only in the third or fourth decimal place. Hence very sensitive optics are required to detect changes in density in gases. Gas flows with changing density are collectively called compressible flows, and compressibility can be produced by temperature variations and/or appreciable Mach number such that $M^2$ is not substantially
smaller than 1. Refractive indices of transparent liquids and solids are much higher than those of gases, so much less optical sensitivity is required to observe refractions in liquids or solids than gases [1].

A light ray passing orthogonally through a change in refractive index will experience a change in phase velocity but will continue traveling in the same direction. If the light ray intersects the change in refractive index obliquely, however, it will bend towards the region with greater \( n \). For a light ray travelling in the \( z \)-direction intersecting a region of varying refractive index the ray curvature is given by

\[
\frac{\partial^2 x}{\partial z^2} = \frac{1}{n} \frac{\partial n}{\partial x} \tag{2}
\]

Here \( x \) is used to represent a direction orthogonal to the ray propagation. Integrating once, the angular ray deflection is

\[
\epsilon = \frac{1}{n} \int \frac{\partial n}{\partial x} \, dz \tag{3}
\]

From Equation 3 we see that ray deflection is dependent not only on the refractive index of the medium but mainly on the gradients in \( n \) orthogonal to the ray propagation direction. Because of the integral in \( z \), deflection is also quite sensitive to the extent of the region of varying density in the \( z \)-direction. These deflections are what are made visible by schlieren and shadowgraph techniques.

### 2.1 Shadowgraph

Shadowgraph techniques are perhaps the simplest methods for visualizing compressible flows. Figure 1 shows a schematic of a typical shadowgraph setup.

![Figure 1: Schematic of a typical shadowgraph setup.](image)

Light from a small source is collimated (made parallel) by a focusing mirror or lens and passes through a test field with varying refractive index. The light source must be small to avoid blur in the image, which is given by \( ld/f_1 \) where \( l \) is the distance of the observation plane from the optical disturbance and depends on the physical size of the camera detector, \( f_1 \) is the focal length of the first focusing mirror or lens, and \( d \) is the size of the light source. The source cannot be too small, however, because very small light sources suffer from a lack of sharpness due to diffraction. The parallel light is then refocused by a second lens or mirror and imaged by a camera. If the second derivative of density is not constant, i.e. \( \partial^2 \rho/\partial y^2 \neq 0 \), the
Figure 2: Schematic showing the refraction of light rays by density fields with density and its first, second, and third derivatives constant. The two-dimensional description is easily extended to three dimensions.

The region where this occurs will appear dark, or as a shadow. Figure 2 illustrates the creation of shadows from a non-constant second derivative in density.

Shadowgraphs are typically used to study shocks and Prandtl-Meyer expansions as both of these flow features create non-constant second derivatives in density. Shadowgraphs are also useful for imaging boundary layers and turbulent flows. Shadowgraphs have an advantage over schlieren setups for such flows because shadowgraph visualization is uniformly sensitive in all directions. Figure 3 shows a typical shadowgraph image.

Figure 3: Shadowgraph image of a bullet in supersonic flow. The bow shock is clearly visible as a dark line, as are other weaker shocks and the turbulent wake behind the bullet.

2.2 Schlieren

Schlieren visualization is similar to the shadowgraph technique, but the primary difference is that while shadowgraphs are sensitive to changes in the second derivative in density, schlieren systems detect changes
to the first derivative in density. A schlieren setup is nearly identical to that of a shadowgraph but with the addition of a knife edge at the focal point of the second lens or mirror as shown in Figure 4. The amount of light blocked by the knife edge is commonly referred to as “cutoff.”

Figure 4: Schematic of a typical schlieren setup.

Undeflected light rays are affected uniformly by the knife edge and the intensity of the image is reduced with increased cutoff. As light passes through a density field with a non-constant first derivative, the light rays are deflected as shown in Figure 2. If light rays are deflected towards the knife edge, the part of the image where those light rays originate from will be darkened more than a part of the image with constant density. Conversely, if light rays are deflected away from the knife edge, that part of the image will appear brighter than unaffected regions of the image. Thus schlieren setups are only sensitive to density gradients normal to the knife edge. An example schlieren image is shown in Figure 5. Notice in this image that the brightness of waves appears reversed in the top and bottom of the image. This is a common feature of many schlieren images because light is being deflected in opposite directions by gradients on the top and bottom of the model.

The primary advantage of schlieren visualization over shadowgraph visualization is that of sensitivity. The sensitivity of a shadowgraph setup depends primarily upon optical path length, which is typically difficult and expensive to modify. The sensitivity of a schlieren setup, however, mainly depends on the amount of cutoff used. Higher cutoff leads to decreased brightness in the image, so increased sensitivity comes at the cost of necessitating a more powerful light source. A related tradeoff exists for the camera. Increased exposure time increases the brightness of the image but reduces the ability to observe transient phenomena like nonsteady waves or turbulence in the flow.

In practice, many variations of the basic schlieren setup described here are used. By far the most common is the Z-type schlieren setup, an almost universal standard. Focusing mirrors provide a much larger field of view for the same price as focusing lenses, so the setup is bent into a “Z” shape to utilize the mirrors. Besides the cost advantage, this also saves space in the laboratory. A Z-type schlieren setup does come at a price, however. The nonlinear alignment of the optics create two effects that distort the image, coma and astigmatism. Coma can be eliminated by careful arrangement and alignment of the focusing mirrors. Astigmatism cannot be fully eliminated, but by using a slit light source aligned with the knife edge it can be minimized until it is no longer visible. For more information on this, see Settles [1]. Other variations typically involve the knife edge, such as using a circular or double knife edge to eliminate the reversal in brightness of conventional schlieren images. Circular knife edges have the added benefit of being sensitive to
density gradients in all directions. A further modification is to use color strip filters instead of a knife edge, resulting in colored schlieren images with the color corresponding to the strength of the change in density gradient.

3 Safety considerations and rules

In the first part of the experiment you will be dealing with an open flame. Be very careful not to accidentally ignite anything besides the sterno heater. The gas used for the jet during this laboratory will come from a bottle filled with compressed gas. The bottle is portable and located in the Cann Lab. The bottle is filled with pressures ranging from 207 to 340 bar (3000 to 5000 psi). Working with gas in that pressure range can be very hazardous. Besides risks to hearing from the release of high pressure gas, gas bottles can also readily become extremely dangerous projectiles if the safety valve is broken or damaged. Be very careful when opening or closing the bottles or any valves connected to the high-pressure line and only do so as directed by your TA. Whenever the bottles need to be changed, do not do it by yourself because mishandling high-pressure gas bottles can be very dangerous. Ask the TA to do it or do it under the TA’s careful supervision. You will also be dealing with gas in a somewhat-enclosed environment, which can be an asphixiation hazard if the gas is left running. That is why you must only run the jet when you are actively taking pictures. Ear protection must be worn any time the jet is in operation.

Here are some general guidelines when performing any sort of experiment:

- Always put on necessary safety gear before interacting with the experimental apparatus
- Never work in the laboratory alone or without notifying someone that you are doing so
- Take the time to THINK carefully before performing any steps with potentially hazardous components

The Cann Lab is shared by several experimental groups and there are multiple active experiments in the Lab, some of which are parts of other Ae104b modules and some of which are research projects. Do not
interfere with the setup of any experiments other than the one for the lab that you are working on, and exercise caution when moving in and out of the enclosures of other experiments. Wear any personal protective equipment required by the other experiments when doing so. The schlieren experiment is sharing space with a currently inactive photoelasticity experiment, but the components for the photoelasticity experiment are not to be touched unless you are told to do so by your TA.

4 Experimental Tasks

The experiment is divided into four separate tasks:

1. Assembly and alignment of the visualization system
2. Visualization of a flame from a sterno heater
3. Visualization of an under-expanded jet
4. Visualization of a phenomena of your choice

4.1 Assembly and alignment

The first task is to assemble and align the schlieren setup. This may seem tedious and might take some time to attain good alignment and focus, but a well-aligned setup will make all the difference. The instructions here are a summary of Chapter 8 of Settles [1]; the reader is referred there for a much more complete guide. Chapter 7 also contains some useful tips.

The primary components for the schlieren setup in this experiment are two 4.25-inch (108 mm) diameter f/10 spherical focusing mirrors. The “f-number” of a mirror is the ratio of the focal length to the diameter of the mirror. These mirrors have a focal length of 45 inches (114 cm). Figure 6 shows a to-scale sketch of a recommended layout for the optics. A functional overall length for the system based on the mirrors and the space available on the optical table is 2 m (78.75”). This length does not need to be exact. A recommended entrance/exit angle for the beams is somewhere between $2\theta = 15^\circ$ and $2\theta = 20^\circ$, as shown in the figure. This also does not need to be exact, but it is crucial that the system is symmetric.

Begin by marking off an open 2 m-length on the optical table with tape. It is best to measure some angle between 15 and 20 degrees relative to the holes on the optical table for this line to make the alignment of other optics easier. Place the two mirrors at opposite ends of the tape, facing each other with the surface of each mirror coincident with an end of the tape. The mirrors will need to be raised off the table in order to make their centerline coincide with the height of the light source. Use some of the plastic samples from the photoelasticity experiment to do this. Mark the center of the tape to designate the test area. Next, prepare a key instrument for alignment: a clean sheet of white paper. To make alignment easier, draw a 4.25”-diameter circle on the paper to designate the size of the test beam. Assemble the light source if necessary. Your TA will give you further instructions on this. Make sure not to exceed the current load of the LED. Measure 45” (the focal length of the mirrors) along a row of holes beginning at the surface of the first mirror and place the light source there and point it at the first mirror.

At this point it may be helpful to dim the lights in the laboratory or turn them off entirely. Carefully adjust the distance of the light source from the mirror until the light beam reflected off the first mirror is a circle with 4.25” diameter irrespective of distance from the mirror. This should be tested using the paper prepared earlier. Using the rotation and tilt controls on the first mirror, point the light beam towards the second mirror such that the entire second mirror is illuminated. Next measure 45” along a row of holes from the second mirror as shown in Figure 6 to form a “Z.” Place the knife-edge at the focus of the second mirror but retract it fully below the beam to allow all the light to pass.
Most schlieren imaging setups send the light directly through a lens and into a camera after the knife edge. This allows the maximum amount of light to be captured by the camera. This requires complete control over the camera and focusing optics. The camera used in this experiment has built-in focusing optics that cannot be removed, so a different technique will be used. An image will be formed on a white screen and the camera will record pictures from the screen image. A good deal of light is lost due to scattering from the screen with this method, but it allows the camera used in this experiment to image the entire field of view and achieve a focused image. The camera is also quite sensitive to light, so imaging a screen avoids saturating the camera when recording shadowgraph images at the expense of some image quality with large amounts of schlieren cutoff.

Position the screen about 12” after the knife-edge and place a bolt in the test region. The bolt can rest on a stack of boxes or anything handy to elevate it into the test region. Next, place a focusing lens between the screen and the knife-edge. Carefully adjust the positions of the screen and the lens until the threads of the bolt become distinguishable and the image formed on the screen is sufficiently large. Secure the lens in place. Angle the screen slightly away from the “Z” and position the camera as shown in Figure 7. Adjust the settings on the camera to manual focus and exposure. Increase the ISO sensitivity to its maximum value. Set the focus to 6” (its shortest focal length) and position the camera such that the word “FOCUS” written on the screen is clearly visible. Then aim the camera at the image on the screen and adjust the zoom until the image nearly fills the field of view of the camera. Adjust the angle of the screen and the positioning of the camera (but not its distance from the screen) until the image is a circle when viewed by the camera. Secure the camera in place.

The steps up to this point may take some time, especially if it is your first time setting up a schlieren system. Do not be afraid to take everything apart and start from scratch if you cannot get a fully-illuminated, well-focused image at the camera. Your images will only be as good as the alignment of your system. Once the system is properly aligned and focused, slowly bring the knife-edge into the beam as you watch the image on the screen. When the knife-edge is perfectly aligned with the focus, the brightness of the image will change uniformly as the knife-edge is moved in and out of the beam. If the image darkens on the same
side as the knife-edge, the knife-edge is too close to the second focusing mirror. If the opposite occurs, it is too far away. Adjust the position of the knife-edge until the image darkens uniformly as the knife-edge is moved into the light beam. Refer to Section 2.2 for a discussion on the amount of light to block. Remember that a shadowgraph image will be produced with zero cutoff.

At this point your schlieren system should be aligned! There are multiple ways to test it, but among the most popular are rubbing one’s hands together and placing a hand in the test area and observing the thermal plume from the hand, breathing into the test region and viewing the gradients created by exhaling, or blowing into the test region with a compressed gas duster. Flames from matches or cigarette lighters work well too. At this point you may also want to practice recording single images and short movies with the camera while adjusting the frame rate and resolution.

4.2 Flame

The first object for visualization will be the flame from a sterno heater. Place the sterno lamp provided under the field of view so that the wick is barely visible at the bottom of the frame. Again, you may need to prop the sterno up using boxes or anything else that is handy. Do not leave the flame burning but only light it when actively recording images/movies; we want the sterno to last through multiple groups’ experiments. Begin by taking single, high-resolution images of the flame over the sterno. Take two schlieren images with horizontal and vertical cutoff and one shadowgraph. Comment on the differences between the images and on any interesting fluid-dynamic phenomena you observe.

Next, set the camera to a frame rate of a few hundred frames per second. The resolution may necessarily be reduced to accomplish this. Record a schlieren movie of the flame for a few seconds. Process the images from the movie using an algorithm of your choice and perform a Fourier analysis to determine the frequency of the flame flicker. Include your code as an appendix to your report. Perform appropriate error analysis to determine an uncertainty in the calculated frequency.
4.3 Under-expanded jet

Once you are finished with the sterno, you will image a jet discharging into air. Assuming isentropic flow, a pressure ratio of about 2 will cause the converging nozzle to choke and the flow at the exit to become sonic. When operating the jet, only open the valve to start flow to set the reservoir pressure and when you are ready to record images. DO NOT leave the flow running, both to conserve gas and to avoid health hazards. Wear ear protection at all times when operating the nozzle. Mount the jet such that the exit of the jet is barely visible on the edge of the field of view of the camera and pointing towards the center of the field of view. Take care not to block other parts of the optical path with components of the jet apparatus.

Choose a reservoir pressure to begin with by adjusting the regulator on the gas cylinder. Use the digital Ashcroft gauge to record the pressure when the gas is flowing. Adjust the cutoff on the schlieren image until the jet is visible. You should be able to see the core of the jet that penetrates a certain distance into the ambient air before turbulent breakdown occurs. This distance is directly dependent on the momentum of the jet, which depends on the reservoir pressure. Take a schlieren image for 10 different reservoir pressures, and make a plot of penetration distance normalized by jet exit diameter versus reservoir pressure normalized by ambient pressure, including horizontal and vertical error bars.

4.4 Your choice

This is the creative portion of the experiment. Find something that you think will produce interesting and/or beautiful schlieren/shadowgraph images or movies and image it using your schlieren setup. There are some suggestions in Chapter 9 of Settles, but you are encouraged to come up with your own original idea. You need not produce any quantitative data in this section, but of course if you can that is a plus. Describe the phenomena and include images or movie frames.

5 Report requirements

The results of the experiment should be documented in a report. The report should include the following:

- Discussion of the theory of schlieren and shadowgraph techniques. You may cite this handout but citation of other sources such as Settles [1] or Chapter 9 of Smits [2] is encouraged.

- Description of the setup in the Cann Laboratory

- Results of each experiment as described in Sections 4.2, 4.3, and 4.4

The structure of the report typically follows the following outline: Abstract $\rightarrow$ Introduction $\rightarrow$ Theory $\rightarrow$ Experimental Setup and Procedure $\rightarrow$ Results and discussion for each sub-experiment $\rightarrow$ Conclusion $\rightarrow$ Appendices. Remember to properly cite any sources used, including this handout.

References
