# GALCIT Ludwieg Tube

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## 1 Principles of supersonic wind tunnels

Supersonic wind tunnels, like their subsonic counterparts, operate by accelerating a high pressure gas through a nozzle and into a test section where the model being tested is placed. Supersonic wind tunnels are generally divided into two categories: continuous and blow-down (sometimes called impulse) facilities. Continuous operation facilities are nearly identical in principle to subsonic tunnels: a compressor increases the pressure of the gas, the gas is accelerated through a nozzle, passes through the test section, is decelerated in a diffuser, and returns to the compressor. Blow-down facilities operate by separating the nozzle and test section from a pressurized section by a diaphragm or valve and then either breaking the diaphragm or opening the valve. The resulting pressure difference drives gas through the nozzle and through the test section, creating supersonic flow as long as the pressure in the driving chamber remains high enough.

Each type has its own advantages and disadvantages. Continuous operation tunnels can perform longduration experiments because they can often operate under steady conditions indefinitely. Because of the high speeds attained in supersonic tunnels, however, the energy consumption from a continuous supersonic wind tunnel is typically very high. Continuous wind tunnels are also typically expensive to maintain because of mechanical components such as the compressor. Blow-down tunnels are by comparison very inexpensive because they often involve no moving parts and the cost associated with an experiment is limited to the cost of gas and single-experiment components as maintenance is seldom needed. Additionally, some blowdown facilities such as shock tunnels and expansion tubes are capable of producing hypervelocity test flow which is impossible for continuous operation wind tunnels because the high reservoir enthalpies required for hypervelocity flow can only be created by nonsteady processes. Blow-down facilities of course have a limited test time, which is often very short (less than 1 second) due to rapid transients associated with high-speed flow. The development of high-speed cameras and transducers has significantly lessened the impact of this disadvantage. For instance, a typical high-speed camera can take several hundred to tens of thousands of high-quality images in the test time of ordinary blow-down facilities, making them well-suited to almost any experiment in high-speed flow.

## 2 GALCIT Ludwieg Tube

A Ludwieg tube is a simple type of blow-down facility that operates somewhat like a shock tube. A diaphragm initially separates a driver tube from a converging-diverging nozzle which feeds into a test section and



Figure 1: GALCIT Ludwieg Tube

dump tank. The driver tube is pressurized until the diaphragm bursts, which produces a weak shock wave propagating into the evacuated nozzle and test section and a nonsteady expansion wave propagating into the driver tube. The expansion wave travels down the driver tube to its end wall and reflects back to the nozzle entrance. During this time there are steady conditions at the nozzle entrance which produce steady flow through the nozzle and steady supersonic flow in the test section. The large volume of the evacuated dump tank ensures that the ambient pressure at the nozzle exit does not increase appreciably during the run time. The run time is limited by the time it takes the nonsteady expansion wave to propagate from one end of the driver tube to the other and back, or approximately twice the length of the driver tube. It is possible to obtain multiple run times from a single diaphragm in a Ludwieg tube. When the reflected nonsteady expansion reaches the nozzle entrance it will reflect back again, and this process will repeat until equilibrium is reached. Each time the expansion runs through the driver tube the pressure is reduced, but as long as it remains high enough for sustaining supersonic flow through the nozzle the tunnel will continue to operate as a Ludwieg tube. This is a common practice in some Ludwieg tube facilities but not at GALCIT.

The GALCIT Ludwieg tube (see Figure 1) has an overall length of 23.5 m with a 29.8 cm driver tube diameter. The tunnel is designed for operation with air as the test gas, although it could in principle be operated with other gases. The driver tube is 17.5 m long, and since nonsteady expansions travel at the local speed of sound of the gas, 340 m/s for air, so the test time is approximately 100 ms. The driver tube is designed for pressures up to 10 bar but has never been operated above 5 bar. A safety release valve at the end of the driver tube prevents the tube from being pressurized above 100 psi (6.9 bar). The GALCIT Ludwieg tube has two nozzles that are interchangeable. One is designed for Mach 2.3 flow, and the other for Mach 4 flow. The Mach 2.3 nozzle can be operated in two configurations, with an upstream diaphragm as described above, or with a downstream diaphragm located at the test section exit. In the downstream mode, a diaphragm breaker allows precise control of free stream conditions and a quieter flow is produced. The Mach 4 nozzle will be used for this experiment and can only operate with an upstream diaphragm. The

Mach 4 nozzle has a 39 cm by 39 cm test section. A section view of the Mach 4 nozzle and test section is shown in Figure 2.



Figure 2: Ludwieg Tube Mach 4 nozzle and test section. The nozzle is instrumented with pressure taps to monitor static pressure.

## 3 Goal of the experiment

The goal of the experiment is to investigate supersonic flow over a wedge at various angles of attack using the Ludwieg tube. We will verify the two-dimensional oblique shock relations and the critical angle at which the oblique shock detaches from the wedge to become locally normal. We will learn basic techniques of high speed flow visualization with a special focus on schlieren and shadowgraph techniques.

# 4 Operation of the Ludwieg Tube

The Ludwieg tube is operated as follows. A diaphragm is cut out and placed between the driver tube and the entrance to the nozzle. The junction between these two is securely closed, and both the driver tube and test section are pumped down by a high-capacity vacuum pump. This takes some time, so diagnostics can be prepared while the tunnel is being evacuated. Once the entire tunnel is at vacuum, it is sealed off, the data acquisition system is armed, and the driver tube is pressurized. At a certain pressure depending on diaphragm thickness, the diaphragm will rupture. The tunnel is then vented. Remember:

- make sure you wear earphones when operating the Ludwieg Tube
- always follow the checklist (in the order specified) when doing an experiment

- always make sure your data acquisition system is triggering properly before each experiment
- never do anything that is not on the checklist unless instructed to do so by your TA

### 5 Flow visualization

The primary diagnostic used in this experiment is visualization of the supersonic flow field. Images are taken with a Vision Research Phantom high-speed camera at several thousand frames per second. Precise camera settings will be determined with your TA. Most methods for visualizing compressible flows depend on the changing density in the flow. The optical index of refraction for a gas is linearly dependent on density, so a non-uniform density field will distort transmitted light waves. This distortion can then be visualized. For a more complete discussion of compressible flow visualization see Chapter 9 of Smits [1].

#### 5.1 Shadowgraph

Shadowgraph techniques are perhaps the simplest methods for visualizing compressible flows. Figure 3 shows a schematic of a typical shadowgraph setup.



Figure 3: Schematic of a typical shadowgraph setup.

Light from a small source is parallelized by a focusing mirror or lens and passes through a test field, such as the test section of a wind tunnel. The light source must be small to avoid blur in the image, which is given by  $ld/f_1$  where l is the distance of the observation plane from the optical disturbance and depends on the physical size of the camera CCD,  $f_1$  is the focal length of the first focusing mirror or lens, and dis the size of the light source. The source cannot be too small, however, because very small light sources suffer from a lack of sharpness due to diffraction. The parallel light is then refocused by a second lens or mirror and imaged by a camera. If the second derivative of density is not constant, i.e.  $\partial^3 \rho / \partial y^3 \neq 0$ , the region where this occurs will appear dark, or as a shadow. Figure 4 illustrates the creation of shadows from a non-constant second derivative in density.

Shadowgraphs are typically used to study shocks and Prandtl-Meyer expansions as both of these flow features create non-constant second derivatives in density. Shadowgraphs are also useful for imaging boundary layers and turbulent flows. Figure 5 shows a typical shadowgraph image.



Figure 4: Schematic showing the refraction of light rays by density fields with density and its first, second, and third derivatives constant. The two-dimensional description is easily extended to three dimensions.



Figure 5: Shadowgraph image of a bullet in supersonic flow. The bow shock is clearly visible as a dark line, as are other weaker shocks and the turbulent wake behind the bullet.

#### 5.2 Schlieren

Schlieren visualization is similar to the shadowgraph technique, but the primary difference is that while shadowgraphs are sensitive to changes in the second derivative in density, schlieren systems detect changes to the first derivative in density. A schlieren setup is nearly identical to that of a shadowgraph but with the addition of a knife edge at the focal point of the second lens or mirror as shown in Figure 6. The amount of light blocked by the knife edge is commonly referred to as "cutoff."

Undeflected light rays are affected uniformly by the knife edge and the intensity of the image is reduced with increased cutoff. As light passes through a density field with a non-constant first derivative, the light rays are deflected as shown in Figure 4. If light rays are deflected towards the knife edge, the part of the image where those light rays originate from will be darkened more than a part of the image with constant density. Conversely, if light rays are deflected away from the knife edge, that part of the image will appear brighter than unaffected regions of the image. Thus schlieren setups are sensitive to density gradients normal



Figure 6: Schematic of a typical schlieren setup.

to the knife edge. An example schlieren image is shown in Figure 7. Notice in this image that the brightness of waves appears reversed in the top and bottom of the image. This is a common feature of many schlieren images because light is being deflected in opposite directions by gradients on the top and bottom of the model.



Figure 7: Schlieren image of a wedge in supersonic flow. Notice the difference in the appearance of shocks and other waves between the schlieren image and the shadowgraph.

The primary advantage of schlieren visualization over shadowgraph visualization is that of sensitivity. The sensitivity of a shadowgraph setup depends primarily upon optical path length, which is typically difficult and expensive to modify. The sensitivity of a schlieren setup, however, mainly depends on the amount of cutoff used. Higher cutoff leads to decreased brightness in the image, so increased sensitivity comes at the cost of necessitating a more powerful light source. A related tradeoff exists for the camera. Increased exposure time increases the brightness of the image but reduces the ability to observe transient phenomena like nonsteady waves or turbulence in the flow.

In practice, many variations of the basic schlieren setup described here are used. By far the most common is the Z-type schlieren setup, an almost universal standard. Focusing mirrors provide a much larger field of view for the same price as focusing lenses, so the setup is bent into a "Z" shape to utilize the mirrors. Besides the cost advantage, this also saves space in the laboratory. A Z-type schlieren setup does come at a price, however. The nonlinear alignment of the optics create two effects that distort the image, coma and astigmatism. Coma can be eliminated by careful arrangement and alignment of the focusing mirrors. Astigmatism cannot be fully eliminated, but by using a slit light source aligned with the knife edge it can be minimized until it is no longer visible. For more information on this, see Settles [2]. Other variations typically involve the knife edge, such as using a circular or double knife edge to eliminate the reversal in brightness of conventional schlieren images. Circular knife edges have the added benefit of being sensitive to density gradients in all directions. A further modification is to use color strip filters instead of a knife edge, resulting in colored schlieren images with the color corresponding to the strength of the change in density gradient.

#### 5.3 Interferometry

A third method for visualization of compressible flows is interferometry. Interferometry has the advantages of being both quantitative and sensitive to changes in density instead of its derivatives. Various setups for interferometry exist and they will not be discussed in detail here. It is sufficient to know that interferometric techniques exist, that they have significant advantages over schlieren and shadowgraph techniques, but that they are often difficult to construct and align and can involve expensive components.

## 6 Auxiliary diagnostics

A number of other diagnostic instruments are used on the GALCIT Ludwieg Tube to determine run conditions. A Kulite XT-190 piezoresistive pressure transducer and an Omega KMQXL-125E-6 K-type thermocouple are installed at the reservoir just upstream of the diaphragm to precisely determine pressure and temperature in the driver tube during the run. Two more piezoresistive pressure transducers can be placed at various locations along the nozzle to determine static pressure at those locations, although these measurements are not usually important. Once pressure and temperature in the nozzle reservoir are known along with the free stream Mach number, all free stream properties can be determined from quasi-1D isentropic relations (see Section 10).

#### 7 Test matrix

In the Ae104 experiments, the Ludwieg tube will be operated with 0.005" polycarbonate diaphragms which result in a burst pressure of approximately 250 kPa (absolute). A total of 10 shots will be performed with a different wedge angle for each experiment. The angle will be varied from 0 degrees to its maximum value of approximately 45 degrees.

## 8 Safety considerations

As a part of this lab, you will be working with high pressure gases when filling the driver tube. Accidental release of high-pressure gas into the laboratory will create shock waves that can cause significant hearing

damage and possibly rupture your eardrums. Furthermore, the velocity of the gas behind the shock wave is strong enough to launch nearby objects (such as tools and wall hangings) across the room, which could result in further injuries. The checklist exists to minimize the possibility of lab accidents. Headphones can save your hearing should accidental venting of high pressure gas occur as it only takes a relatively weak shock wave to damage your hearing. To give you an idea of the pressures you will be working with, here are some numbers to keep in mind. The numbers are given in terms of overpressure, which is the pressure in excess of atmospheric pressure. You eardrums can rupture at overpressures of 0.34 bar[3] and 50% of exposed eardrums rupture at overpressures of 1 bar[3]. Furthermore, lung damage can occur at overpressures in excess of 1 bar.

Additionally, the Ludwieg tube is a large and heavy piece of equipment. Do not overly strain yourself attempting to move the tunnel around. Help each other with heavy lifting, and don't be afraid to ask the TA or others for help. Be careful when using tools not to injure yourself or the facility.

You will notice that the rail supporting the nozzle and test section appears to have multiple pieces that were added at different times. The nozzle was formerly supported only in one location by a small carrier and the rail only extended as far as the blue beam in Figure 2. This means that unless the nozzle was connected at one of the two junctions with either the dump tank or driver tube at all times, it was not stable. At one point, a student using the Ludwieg Tube was not THINKing (see the following list) about what he was doing and disconnected both junctions simultaneously. The nozzle, 1000 pounds of aluminum, crashed into the floor and could have seriously injured or even killed the student. Fortunately the student was not hurt, and modifications were made to the nozzle to prevent this from happening again. This example shows, however, that even a very simple machine can be very dangerous if it is not given proper respect. When performing experiments of any kind, take appropriate precautions to keep yourself safe.

It is critical that you observe the following rules while performing this experiment:

- 1. Follow the checklist at ALL times. You should place a checkmark next to each step immediately after you do it.
- 2. THINK about each step that you are about to perform. Don't blindly follow the checklist.
- 3. Wear ear protection whenever working with gas at a different pressure than ambient pressure. Headphones must be worn from the moment that the Ludwieg tube is evacuated until after pressure equilibrium has been reached after the experiment.

## 9 Gas Fill Bottles

The gas used to fill the driver tube during this laboratory will come from bottles filled with compressed gas. The bottles are portable and located in the Ludwieg tube lab. All the bottles are filled with pressures ranging from 207 to 340 bar (3000 to 5000 psi). Working with gas in that pressure range can be very hazardous. Besides risks to hearing from the release of high pressure gas, gas bottles can also readily become extremely dangerous projectiles if the safety valve is broken or damaged. Be very careful when opening or closing the bottles or any valves connected to the high-pressure line and only do so when directed by the checklist. Whenever the bottles need to be changed, do not do it by yourself because mishandling high-pressure gas bottles can be very dangerous. Ask the TA to do it or do it under the TA's careful supervision.

## 10 Quasi-one-dimensional isentropic flow

Nozzle flow is often treated as quasi-one-dimensional because gradients in the streamwise direction are generally much larger than in other directions. It can also be treated as isentropic because entropy is only generated in the very thin boundary layers near the walls of the nozzle, assuming there are no shocks present inside the nozzle. Because the flow is taken to be frictionless, steady nozzle flow follows the 1D Euler equations:

$$d(\rho u A) = 0 \tag{1}$$

$$dp + \rho u du = 0 \tag{2}$$

$$dh + udu = 0 \tag{3}$$

$$ds = 0 \tag{4}$$

The continuity and energy equations can be integrated directly:

$$\rho u A = \dot{m} = \text{const} \tag{5}$$

$$h + \frac{u^2}{2} = h_t = \text{const} \tag{6}$$

Since the flow is isentropic, we know from the definition of the speed of sound that

$$dp = a^2 d\rho \tag{7}$$

This can be combined with the momentum equation to give

$$\frac{d\rho}{\rho} = -M^2 \frac{du}{u} \tag{8}$$

Which in turn can be combined with continuity to relate velocity to area:

$$\frac{du}{u} = \frac{1}{M^2 - 1} \frac{dA}{A} \tag{9}$$

This reveals the familiar result that subsonic flow accelerates in a converging channel and supersonic flow accelerates in a diverging channel. For a perfect gas, relations for temperature, pressure, density, and area can be related directly to the Mach number. These are the so-called isentropic flow relations:

$$\frac{h_t}{h} = \frac{T_t}{T} = 1 + \frac{\gamma - 1}{2}M^2$$
(10)

$$\frac{\rho_t}{\rho} = \left(1 + \frac{\gamma - 1}{2}M^2\right)^{\frac{1}{\gamma - 1}} \tag{11}$$

$$\frac{p_t}{p} = \left(1 + \frac{\gamma - 1}{2}M^2\right)^{\frac{\gamma}{\gamma - 1}} \tag{12}$$

$$\frac{u}{a_t} = \left(\frac{M^2}{1 + \frac{\gamma - 1}{2}M^2}\right)^{\frac{1}{2}}$$
(13)

$$\frac{A}{A^*} = \frac{1}{M} \left[ \frac{2}{\gamma+1} \left( 1 + \frac{\gamma-1}{2} M^2 \right) \right]^{\frac{\gamma+1}{2(\gamma-1)}}$$
(14)

Here the subscript t indicates total or stagnation conditions and \* indicates sonic conditions. It is important to note that the equation for enthalpy or temperature does not require the flow to be isentropic, only that it is adiabatic because it follows directly from the energy equation. The other relations can be shown to follow directly from this one using the familiar isentropic relationships for thermodynamic variables for a perfect gas.

It is useful to briefly examine the high Mach number limit  $(M \to \infty)$  for fixed stagnation conditions:

$$h, a, \rho, p \to 0 \tag{15}$$

$$u \to \sqrt{\frac{2}{\gamma - 1}} a_t = \sqrt{2h_t} \tag{16}$$

$$\frac{A}{A^*} \to \infty \tag{17}$$

We notice that although it is possible to increase Mach number indefinitely by increasing the area ratio, the velocity approaches a finite limit. The perfect gas assumption will also break down as the gas approaches condensation as the temperature is decreased.

#### 11 Supersonic nozzle flow

Because of the implications of Equation 9, a supersonic nozzle must have a converging section through which the flow is accelerated from rest to sonic conditions followed by a diverging section where the flow becomes supersonic. This is the familiar converging-diverging or de Laval nozzle. Figure 8 shows the progression A  $\rightarrow$  B  $\rightarrow$  C that takes place as the exit pressure  $p_e$  is decreased starting at  $p_t$  with  $p_t$  held constant. It can be observed from the isentropic flow relation for pressure that there are two conditions for isentropic flow throughout the nozzle and sonic flow at the throat, or point of minimum area. One of these, the condition just between conditions A and B, will have isentropic subsonic flow throughout just like condition A. The other case corresponds to condition C where the pressure at the exit of the nozzle is precisely matched to the exit pressure  $p_e$ . This is commonly referred to as the perfectly expanded condition.

Condition B, which has subsonic flow at the nozzle exit but supersonic flow just past the throat, corresponds to a much higher exit pressure than the perfectly expanded case. A normal shock will appear inside the diverging section of the nozzle, causing the supersonic flow to become subsonic. Past the normal shock, the subsonic flow will decelerate in the diverging nozzle and the pressure will increase. The precise location of the shock is determined by the exit pressure. As the exit pressure is lowered, the normal shock will proceed towards the nozzle exit and will eventually be precisely at the exit. As the exit pressure is lowered further, a smaller increase in pressure than that generated by a normal shock is required to match the pressure. This cannot be achieved with one-dimensional flow, and oblique waves will form at the exit. The oblique waves will become weaker as  $p_e$  is lowered further until they disappear and the perfectly expanded condition is reached. If  $p_e$  is decreased below the perfectly expanded condition, Prandtl-Meyer waves will form at the exit of the nozzle to further expand the flow to match  $p_e$ .

## 12 Oblique shock waves

Because information cannot propagate upstream in supersonic flow, in order for flow to turn around obstacles waves must form. For a sharp obstacle with a positive angle such as a wedge or a cone, an oblique shock wave emanates from the leading edge or tip. Oblique shock waves follow the same relations as normal shock waves except they only affect flow normal to the wave front. The component of velocity parallel to the wave is unaffected. This means that flow can be, and often is, supersonic downstream of oblique shock waves. This is in sharp contrast with normal shock waves. Figure 9 shows the notation commonly used to describe



Figure 8: Pressure profiles in isentropic nozzle flow as the exit pressure is reduced.



Figure 9: Oblique shock wave notation.

oblique shock waves. The incoming condition is labeled 1, the flow processed by the shock is condition 2, the flow turning angle is  $\theta$ , and the wave angle is  $\beta$ .

The preservation of the velocity component parallel to the shock front is given by geometry to be

$$u_1 \cos \beta = u_2 \cos(\beta - \theta) \tag{18}$$

Jump conditions apply to the normal component only, which means that they are identical to the jump conditions for normal shocks except that  $M_1$  is replaced by  $M_1 \sin \beta$  and  $M_2$  is replaced by  $M_2 \sin(\beta - \theta)$ . This gives the following:

$$\frac{p_2}{n_1} = 1 + \frac{2\gamma}{\gamma + 1} (M_1^2 \sin^2 \beta - 1)$$
(19)

$$\frac{\rho_2}{\rho_1} = \frac{M_1^2 \sin^2 \beta}{1 + \frac{\gamma - 1}{\gamma + 1} (M_1^2 \sin^2 \beta - 1)}$$
(20)

$$M_2^2 \sin^2(\beta - \theta) = \frac{1 + \frac{\gamma - 1}{\gamma + 1} (M_1^2 \sin^2 \beta - 1)}{1 + \frac{2\gamma}{2\gamma + 1} (M_1^2 \sin^2 \beta - 1)}$$
(21)

Equation 18 can be combined with the continuity equation for oblique waves,

$$\rho_1 u_1 \sin \beta = \rho_2 u_2 \sin(\beta - \theta) \tag{22}$$

to give a relation for  $\beta$ .

$$\rho_1 \tan \beta = \rho_2 \tan(\beta - \theta) \tag{23}$$

By using the identity for the tangent of a difference of two angles, we derive the  $\beta - \theta - M$  relationship.

$$\tan \theta = \frac{1}{\tan \beta} \frac{M_1^2 \sin^2 \beta - 1}{\frac{\gamma + 1}{2} M_1^2 - (M_1^2 \sin^2 \beta - 1)}$$
(24)

A graph of the  $\beta - \theta - M$  relationship is called the oblique shock polar, shown in Figure 10. Two conclusions can be made immediately from Equation 24. First,  $\theta$  can be zero if  $\beta = \sin^{-1} \frac{1}{M_1} = \mu$ , the Mach angle. This is the angle of an isentropic wave emanating from an infinitesimal disturbance, also called a Mach wave.  $\theta$  can also be zero if  $\beta = \frac{\pi}{2}$ , which corresponds to a normal shock. Secondly, for a given turning angle  $\theta$ , two solutions exist for the wave angle  $\beta$ . The larger value of  $\beta$  is called the strong shock solution, and the smaller value fo  $\beta$  corresponds to the weak shock solution. Flow downstream of a strong shock will be subsonic, and flow downstream of a weak shock is supersonic. The weak shock solution is most often observed in external flows, such as flow past objects in free flight or in wind tunnels, while strong shocks typically occur only in confined flows such as inside ducts or engine inlets where strong pressure gradients exist.

From Figure 10, we can observe other properties of oblique shocks that are not immediately apparent from the  $\beta - \theta - M$  relationship. First, there is a maximum turning angle  $\theta_{\max}$  for which an oblique shock can turn the flow for a given Mach number. This angle increases with increasing Mach number. If the turning angle is larger than this maximum value, the oblique shock detaches from the body and the body behaves like a bluff body where the physical size of the body becomes important and influences the flow, including the shock detachment distance. Furthermore, we notice that there is a "sonic line" where  $M_2 = 1$ , the condition for which happens to be close to that for  $\theta = \theta_{\max}$ . By definition, this line separates weak shocks from strong shocks. Finally, there is a finite limit for the  $\beta - \theta$  curve as  $M_1 \to \infty$ . This is called the hypersonic limit.

#### 13 Non-ideal considerations

In the development of the theory for Sections 10 and 11, several assumptions were made to make the analysis tractable. Non-ideal effects that are ignored by making these assumptions must be considered. First, the gas



Figure 10: Oblique shock polar for  $\gamma = 1.4$ 

is assumed to be ideal and calorically perfect. The pressure is low enough in all flow regimes in the Ludwieg tube for the ideal gas assumption to be valid. Furthermore, the temperature stays between 65 and 300 K, well within the bounds for considering air to be a perfect gas. Vibrational energy modes are not excited at these temperatures, and dissociation/recombination reactions do not occur at an appreciable level. Thus the perfect gas assumption is valid.

Second, the flow is assumed to be isentropic. Since there is no heat addition to the flow, entropy is only added via friction in the boundary layers along the nozzle walls. These boundary layers can be shown to be very thin and do not affect the freestream flow, so the isentropic assumption is valid for determining free stream properties. The boundary layers must be considered, however, when calculating the nozzle area ratio. The isentropic area experienced by the flow is the area inside the nozzle minus the displacement thickness of the boundary layers. This effect is dealt with by nozzle designers iteratively. Flow is calculated assuming inviscid flow with the nozzle only, then the boundary layer displacement thickness is computed at each point in the nozzle. The flow is calculated again with the effect of the displacement thickness, and a new displacement thickness is calculated. This procedure is repeated until convergence is reached. In this way the desired exit Mach number can be achieved while considering the displacement thickness effect of boundary layers inside the nozzle.

Third, an assumption was made that the flow through the nozzle is one-dimensional. Of course if the area is changing as a function of streamwise distance then the flow is not exactly one-dimensional. In order

for the supersonic flow in the diverging section of the nozzle to turn in response to the contour of the nozzle walls, waves must be present. If the contour is not well-designed, oblique shocks will form in the nozzle. In order for the flow through the nozzle to remain isentropic, the nozzle must be designed carefully while considering the generation of waves. This is done with the method of characteristics, which is described in detail in various compressible flow textbooks and will not be discussed here. It is sufficient to know that the Mach 4 nozzle in the GALCIT Ludwieg Tube was designed using such a method such that it produces approximately isentropic flow throughout the nozzle and nearly uniform flow at the exit.



Figure 11: Approximate location of test rhombus in the GALCIT Ludwieg Tube

This region of uniform flow is limited, however to the so-called "test rhombus", which is a rhomboid formed by the positive and negative characteristic waves emanating from the nozzle exit. These characteristics are Mach waves that carry the information that the flow has exited the nozzle. Flow is uniform inside the test rhombus, but will not necessarily be uniform outside of it as the flow responds to the information carried by the characteristic waves. Figure 11 shows the approximate location of the test rhombus for the GALCIT Ludwieg Tube. Care must be taken to ensure that the entirety of the test article lies inside of the test rhombus.

Finally, various flow regimes are described for supersonic nozzle flow in Section 11. For a given area ratio, however, only one particular pressure ratio will correspond to the perfectly expanded condition. This precise ratio is very difficult to attain in practice. However, since the only region of the flow that is important for wind tunnels is the region inside of the test rhombus, the perfectly expanded condition is identical to the underexpanded condition which has Prandtl-Meyer waves at the exit. Since the first Prandtl-Meyer wave is a Mach wave, the beginning of the Prandtl-Meyer expansion fan will coincide with the boundary of the test rhombus. Underexpanded flow is much easier to attain in practice since it only requires a very low back pressure at the nozzle exit. This is achieved by reducing the pressure in the test section to a partial vacuum.

It is mentioned in Section 2 that Ludwieg tubes can potentially operate several times for a single diaphragm burst as long as the pressure ratio remains high enough that the nozzle is underexpanded. It turns out that this is not the case for the Mach 4 nozzle on the GALCIT Ludwieg Tube with 0.005" diaphragms; after the first test time the pressure ratio becomes such that the flow is overexpanded and oblique shocks penetrate into the test rhombus. This discussion also demonstrates why the Mach 4 nozzle can only be operated with an upstream diaphragm. If the diaphragm is in the downstream position, the low pressure produced by a nonsteady expansion wave in the test section is not low enough to start isentropic supersonic flow through the nozzle. In this case unsteady processes occur until the pressure is equilibrated in the tunnel and subsonic flow occurs throughout. Issues related to the starting and unstarting of supersonic nozzles are well-documented in compressible flow textbooks and must be considered in all supersonic wind tunnels.

# 14 Report requirements

The results of the experiment should be documented in a report. The report should include the following:

- Description of the facility.
- Calculate the velocity at which the nozzle and test section would hit the floor of the laboratory if it were to fall off of its supporting rails (as described in the incident in Section 8). How fast would you have to swing a sledgehammer to generate the same impulse?
- Determine and describe an additional safety concern in the Ludwieg tube laboratory not described in this handout.
- Table of run conditions, including results, i.e., the table should contain shot number, wedge angle, and shock angle.
- Use the measured wedge angles and shock angles to construct a curve from the shock polar. Graph this curve. Do you observe weak shocks or strong shocks?
- Experimentally determine the freestream Mach number from the  $\beta \theta M$  relationship
- From this value of M, predict the maximum turning angle. Verify this experimentally (use 2-3 shots of your allotted 10 to do this)
- Provide schlieren and/or shadowgraph images of representative cases
- Error analysis-qualitatively discuss what you perceive to be the principle errors in the experimental measurements, **also** provide a quantitative analysis of the experimental error in measuring the wedge and shock angles and propagate this error into the calculated freestream Mach number

The structure of the report typically follows the following outline: Abstract  $\rightarrow$  Introduction  $\rightarrow$  Theory  $\rightarrow$  Experimental Setup and Procedure  $\rightarrow$  Results  $\rightarrow$  Discussion  $\rightarrow$  Error Analysis  $\rightarrow$  Conclusion  $\rightarrow$  Table of Run Conditions. Remember to properly cite any sources used, including this handout.

## References

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